Ph.D. Course on *Vorticity, Vortical Flows and Vortex-Induced Vibrations*
Technical University of Denmark, Copenhagen, Denmark
vortex.compute.dtu.dk
August 26-30, 2019

Vorticity Generation 1: Introduction and Key Concepts

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FLAIR
Fluids Laboratory for Aeronautical and Industrial Research

Course Schedule

Monday August 26	Tuesday August 27	Wednesday August 28	Thursday August 29	Friday August 30
9.00 – 9.45 Introduction (KH) Lecture 1 (KH) Vorticity Generation I	9.00 – 9.45 Lecture 5 (KH) Swirling Flow Instabilities	9.00 – 9.45 Lecture 9 (KH) Bluff Body Wakes I	9.00 – 9.45 Lecture 12 (KH) Fluid-Structure Interactions I	9.00 – 9.45 Lecture 15 (JS) Aerodynamics of Wind Turbines I
9.45 – 10.00 Break	9.45 – 10.00 Break	9.45 – 10.00 Break	9.45 – 10.00 Break	9.45 – 10.00 Break
10.00 – 10.45 Lecture 2 (KH) Vorticity Generation II	10.00 – 10.45 Lecture 6 (KH) Vortex Breakdown	10.00 – 10.45 Lecture 10 (KH) Bluff Body Wakes II	10.00 – 10.45 Lecture 13 (KH) Fluid-Structure Interactions II	10.00 – 10.45 Lecture 16 (JS) Aerodynamics of Wind Turbines II
10.45 – 11.00 Break	10.45 – 11.00 Break	10.45 – 11.00 Break	10.45 – 11.00 Break	10.45 – 11.00 Break
11.00 – 11.45 Lecture 3 Vortex Filaments (TL)	11.00 – 11.45 Lecture 7 (MB) Vortex Dynamics I	11.00 – 11.45 Lecture 11 (KH) Vehicle Aerodynamics	11.00 – 11.45 Lecture 14 (KH) Fluid-Structure Interactions III	11.00 – 11.45 Lecture 17 (KH) Concluding Lecture
11.45 – 13.00 Lunch	11.45 – 13.00 Lunch	11.45 – 13.00 Lunch	11.45 – 13.00 Lunch	11.45 – 13.00 Lunch
13.00 – 13.45 Lecture 4 Long and Short Wave Instabilities (TL)	13.00 – 13.45 Lecture 8 (MB) Vortex Dynamics II	13.00 – Social programme and dinner	13.00 – 14.00 Project discussions	13.00 – 14.00 Project discussions
13.45 – 14.00 Break	13.45 – 14.00 Break			
14.00 – 14.45 Project presentation	14.00 – 14.45 Project discussions			

General course objectives

Vorticity is a measure of the spinning motion of a fluid. Understanding the sources, dynamics and structure of vorticity, in particular the interaction of vortices, is of key importance in fluid mechanics. It is the purpose of the course to give the students an overview of recent research in this field.

Learning objectives

A student who has met the objectives of the course will be able to:

- understand the mathematical definition and the physical meaning of vorticity
- understand how vorticity is generated and be able to analyze the vorticity flux near boundaries and interfaces
- understand the dynamics of vortex filaments, be able to perform a simple stability analysis
- understand the concept of vortex breakdown and indentify it in various engineering situations
- understand the generation of vortices in the wake of bluff bodies and be able to identify different types of vortex wakes
- perform a basic analysis of the topology of two-dimensional flow, based on dynamical systems theory and experimental or computational data
- understand how vortices interact with solid bodies, how vibrations are induced and how to model this interaction mathematically
- understand the generation of vortices behind wind turbines, and how this is modelled

Aims of the Course

Participants will:

- Gain an introduction to various (but not all!) aspects of vorticity and vortices

- Learn of recent research in vortex dynamics, bluff body flows, flow induced vibrations, wind turbines, vortex breakdown

- Practise some techniques of coding (Python/Anaconda), flow analysis (PIV, Stability) and hands-on simple construction and experiments in anemometry, flow-induced vibration, and turbines.

Lecture Reading Material

Batchelor, G. K., An Introduction to Fluid Dynamics, Cambridge University Press, 1967.
Holmén, V. Methods for Vortex Identification. Master's Thesis, Lund University, Lund, Sweden, 2012. Available online: http://lup.lub.lu.se/student-papers/record/3241710
Lighthill, M. J., Laminar Boundary Layers (ed. L. Rosenhead), Oxford University Press, 1963.
Morton, B. R., The generation and decay of vorticity, Geophysical & Astrophysical Fluid Dynamics, 28:3, 277 - 308, 1984.

Lecture Objectives

Learn:

The (vector) definition of vorticity cf angular momentum

The usefulness of considering vorticity (and circulation)

Ways of defining vortices

How vorticity is generated at boundaries, transported and its conservation

Extension to three dimensions

Key Questions

Q1: What is vorticity (and circulation)?

Q2: Why should we worry about vorticity?

Q3: What is a vortex?

Q4: How is vorticity generated?

Q5: How is vorticity transported, and how does it decay?

Q6: Is vorticity conserved?

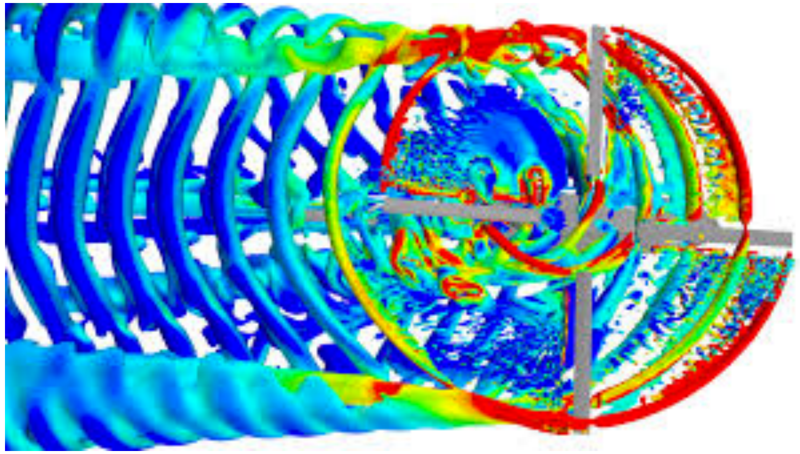
Q1: What is vorticity?

Vorticity is mathematically defined as the curl of the velocity field and is hence a measure of local rotation of the fluid. This definition makes it a vector quantity.

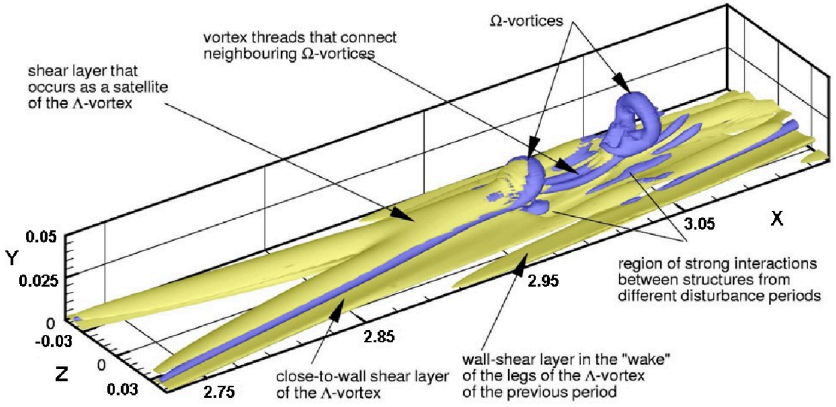
$$\boldsymbol{\omega}(\mathbf{r},t) = \nabla \times \mathbf{v}(\mathbf{r},t)$$

Qualitatively, we usually associate regions of spinning fluid with vorticity, which we term vortices:

Wing tip vorticity
(aviation.stackexchange.com



Vorticity in a rotor wake
(Siemens)



Boundary layer vorticity
Ulrich Rist, *DOI:10.2514/6.2012-84*

Vorticity is local rotation – different to Angular Momentum

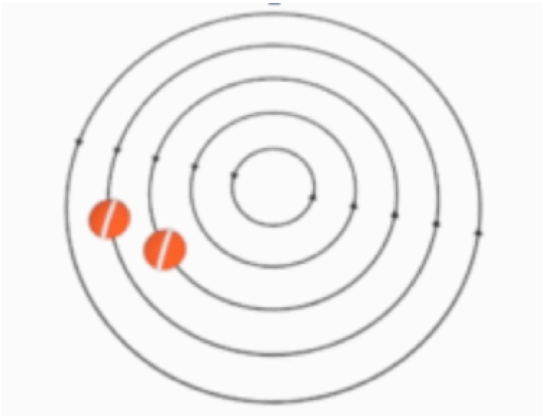
Vorticity is a local "rotation", independent of fluid density

Angular momentum is rotation about a fixed origin (non-local) and density dependent

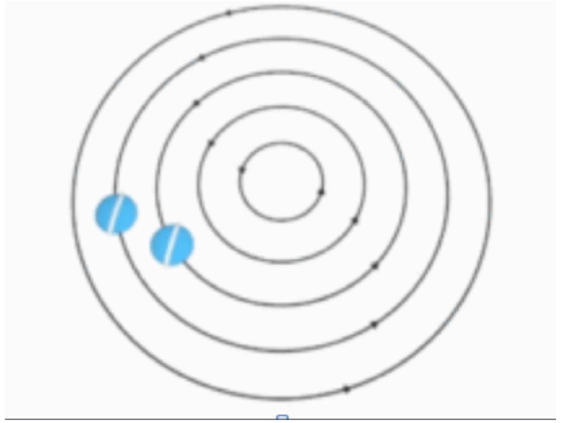
$$\boldsymbol{\omega}(\mathbf{r},t) = \nabla \times \mathbf{v}(\mathbf{r},t)$$

$$\mathbf{L} = \mathbf{r} \times m\mathbf{v}(\mathbf{r}, t)$$

Irrotational flow



Rotational flow



Steady – no diffusion of vorticity or momentum

Angular momentum uniform,
$\omega = 0$ for $r \neq 0$,
$v = \Gamma/(2\pi r)$, circulation = Γ

en.wikipedia.org/wiki/Vortex

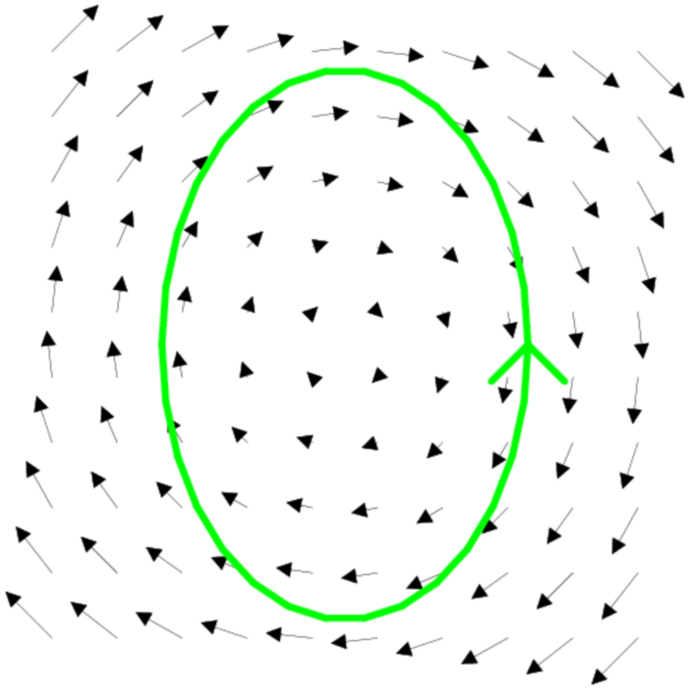
Solid body rotation, no shear,
$\omega = 2\Omega$,
$v = \Omega r$

Circulation

Circulation: line integral of tangential velocity around a closed loop or sum of vorticity over an area A (Stokes' theorem)

Vorticity is therefore circulation density

$$\Gamma = \oint_C \mathbf{V} \cdot d\mathbf{l} \quad = \int_A \omega \, \mathrm{d}A$$



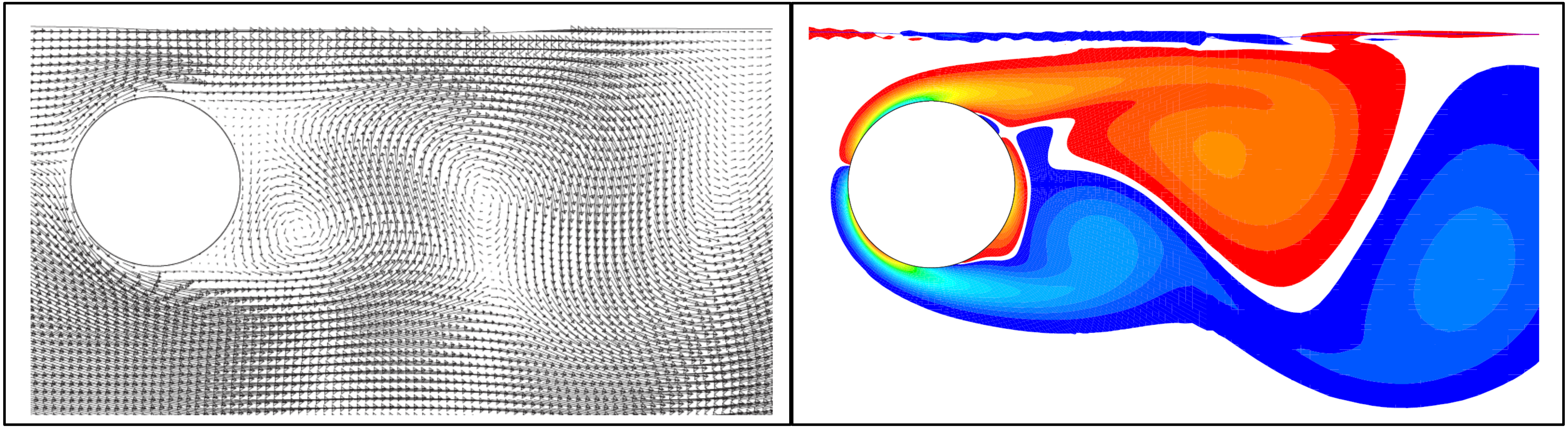
mathinsight.org/line_integral_circulation

Q2: Why should we worry about vorticity?

www.youtube.com/watch?v=Ee4-xtX9jUQ

Flow features are more clearly visualised and understood in terms of vorticity

"Vortices are the sinews and muscles of fluid motions" – Kuchemann (1965)

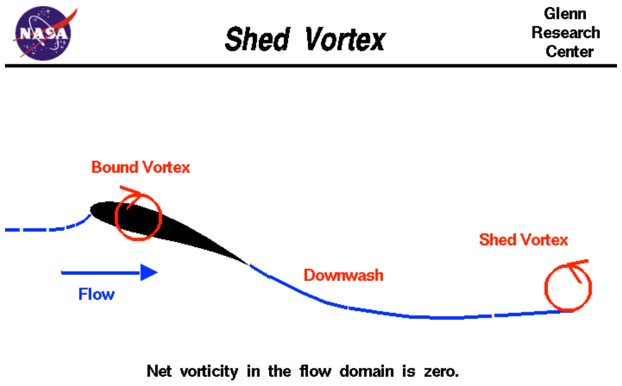


Velocity and vorticity fields for flow past a cylinder near a free surface (Reichl 2002)

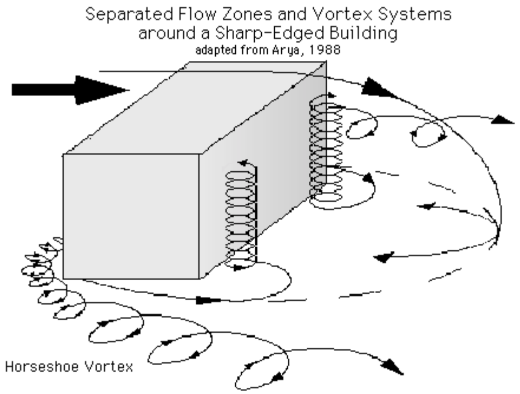
Vorticity field is **independent** of the Galilean or inertial frame of reference (velocity field is not)

Vortices appear in many important flows

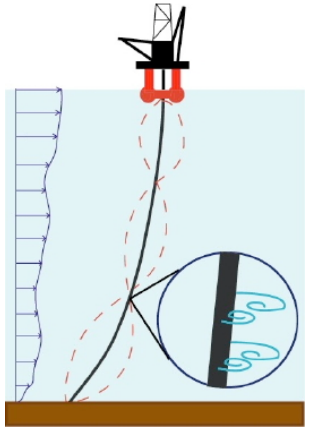
Provides lift for aircraft……………………but constrains flight schedules



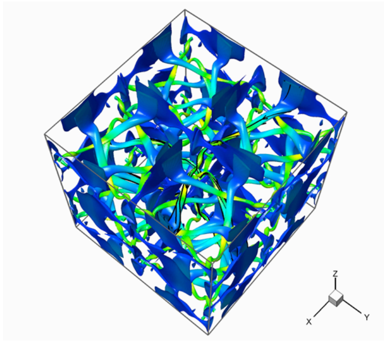
Wind loads on buildings



Vortex induced vibrations



Turbulence



Aircraft Vortices

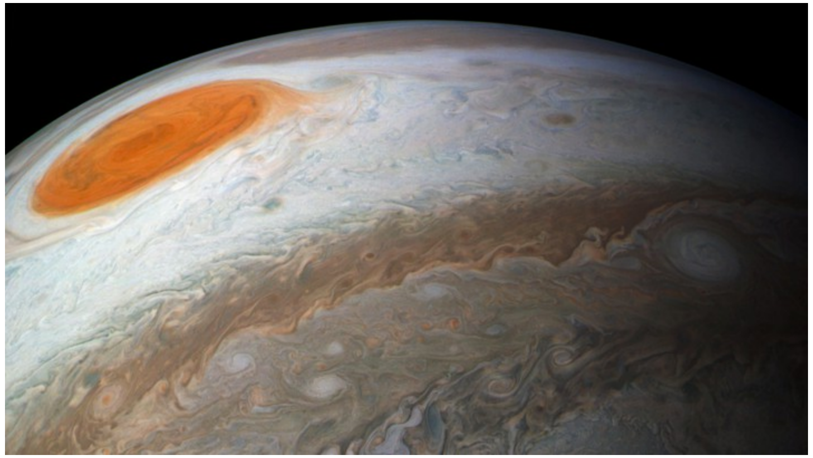
https://www.youtube.com/watch?v=dfY5ZQDzC5s

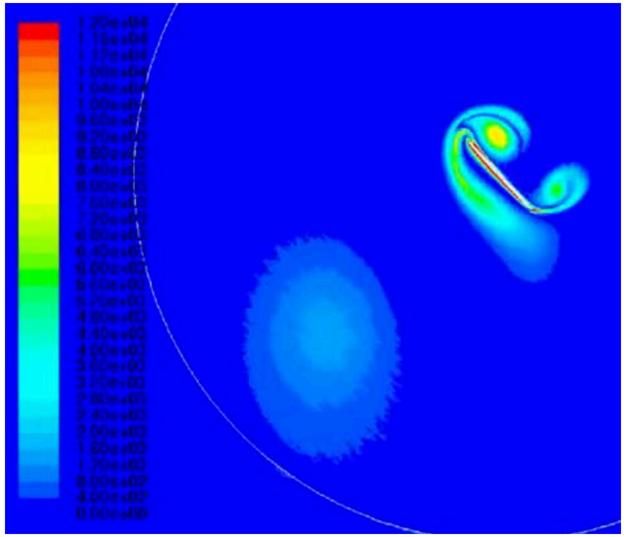
Q3: What is a vortex?

We qualitatively recognise a vortex as a vorticity structure that retains a degree of coherence with time as it is advected by the flow.

However, due to viscosity, there is no clear boundary to a vortex.

Plotting the vorticity contours shows where vorticity is, but does not show where the boundary of a vortex lies.





https://takhtesefid.org/videos/hq462809304165.mp4

Methods of identifying a vortex based on the Velocity Gradient tensor

The velocity gradient tensor D_{ij} can be split into symmetric (e_{ij}) and antisymmetric (ω_{ij}) tensors:

$$D_{ij} \equiv \frac{\partial U_j}{\partial x_i} \equiv e_{ij} + \omega_{ij} = e_{ij} + \frac{1}{2}\epsilon_{ijk}\omega_k$$

where

Rate of Strain Tensor: $e_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)$

Axisymmetric Rotation Tensor: $\omega_{ij} = \frac{1}{2}\left(\frac{\partial U_i}{\partial x_j} - \frac{\partial U_j}{\partial x_i}\right)$

Vorticity Vector: $\omega_k = \epsilon_{kmn}\frac{\partial U_n}{\partial x_m}$

The characteristic equation for D_{ij} is given by:

$$|D_{ij} - \lambda I| = \lambda^3 + P\,\lambda^2 + Q\,\lambda + R = 0$$

where P, Q and R are the three invariants of the velocity gradient tensor.

Using the decomposition into symmetric and anti-symmetric parts these invariants can be expressed as follows:

$$P = -tr(\bar{D})$$

$$Q = \frac{1}{2}(tr(\bar{D})^2 - tr(\bar{D}^2)) = \frac{1}{2}||\bar{\Omega}||^2 - ||\bar{S}||^2$$

$$R = -det(\bar{D})$$

The discriminant of the characteristic equation is:

$$\Delta = \left(\frac{Q}{3}\right)^3 + \left(\frac{R}{2}\right)^2$$

Q-criterion; Δ criterion

Q-criterion

Q represents the local balance between shear rate strain and vorticity magnitude.

Vortices are defined as areas where $Q > 0$

i.e., where the vorticity magnitude is greater than the magnitude of the rate-of-strain.

Δ-criterion

This criterion defines vortices as "regions in which the eigenvalues of D_{ij} are complex and the streamline pattern is spiralling or closed".

For eigenvalues to be complex, the disciminant must be positive:

$$\Delta = \left(\frac{Q}{3}\right)^3 + \left(\frac{R}{2}\right)^2 > 0 \quad \text{(valid for incompressible flows: } P = 0\text{)}.$$

λ_2 criterion

Note that in planar flows, the 3 methods above are equivalent.

λ_2 criterion

Seeks pressure minimum but removes effects from unsteady straining and viscosity by discarding these terms.

Taking gradient of Navier-Stokes equations:

$a_{i,j} = -\frac{1}{\rho} p_{ij} + \nu u_{i,jkk}$ where $a_{i,j}$ is the acceleration gradient and p_{ij} is symmetric.

Decomposing $a_{i,j}$ into symmetric and antisymmetric parts give the vorticity transport equation as the antisymmetric part, and the symmetric part:

$$\frac{DS_{ij}}{Dt} - \nu S_{ij,kk} + \Omega_{ik}\Omega_{kj} + S_{ik}S_{kj} = -\frac{1}{\rho} p_{ij}$$

The first 2 terms on LHS represent unsteady irrotational straining and viscous effects, respectively.

Therefore, only $S^2 + \Omega^2$ is considered to determine if there is a local pressure minimum that entails a vortex.

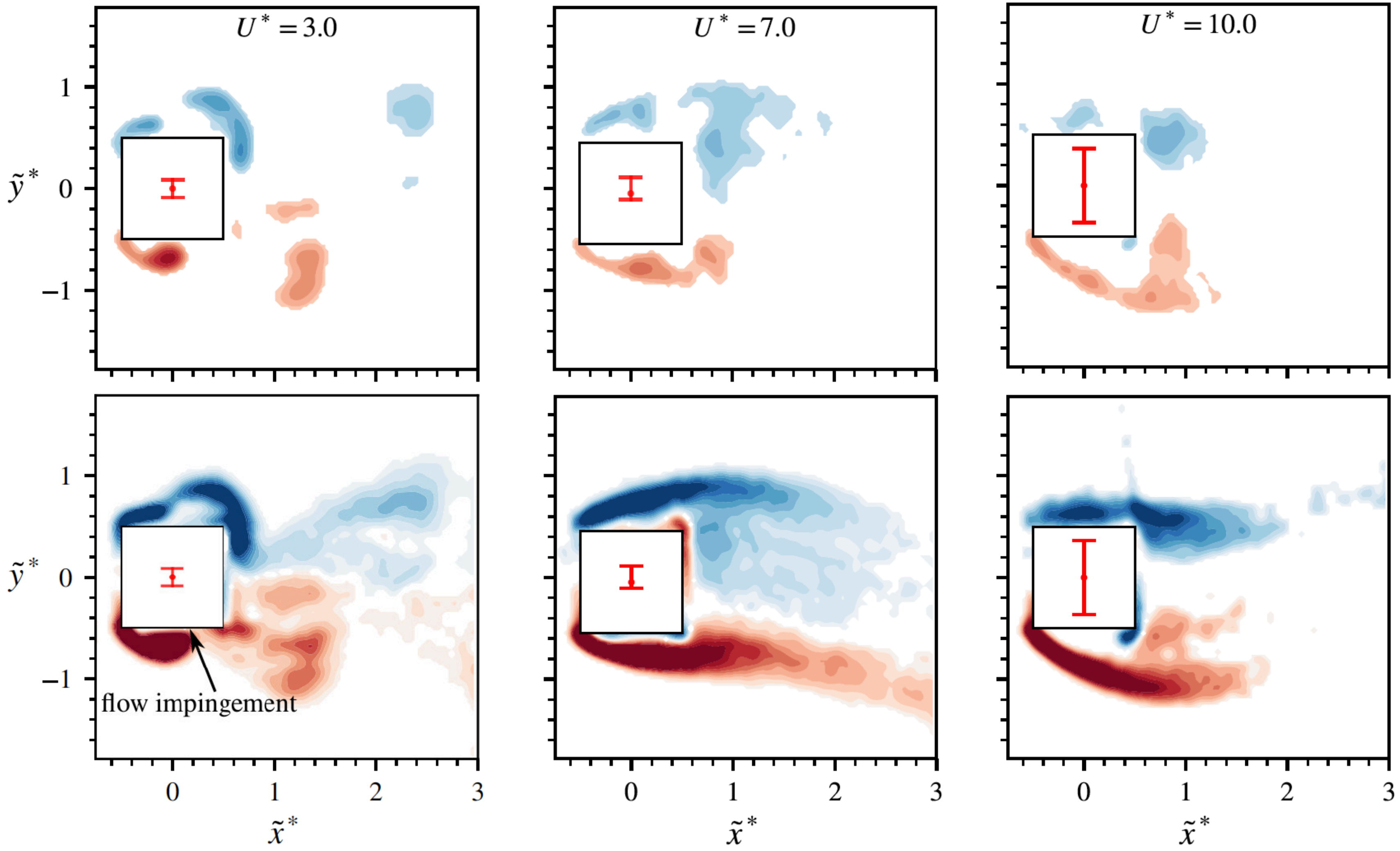
A vortex is defined as "a connected region with 2 negative eigenvalues of $S^2 + \Omega^2$, which is symmetric and therefore has real eigenvalues only. (Jeong & Hussain, 1995).

By ordering the eigenvalues $\lambda_1 < \lambda_2 < \lambda_3$, the definition is equivalent to requiring $\lambda_2 < 0$.

The isosurfaces are usually visualised for different values of $-\lambda_2$.

Other Criteria

There are others such as Swirling Strength: see Methods for Vortex Identification, Vivianne Holmén, 2012)



Swirling Strength (tries to eliminate shear layers)

Vorticity Contours

Figure 2: Comparison between vorticity (in the bottom row) and swirling strength (in the top row) contours at $U^* = 3.0$ (left column), 7.0 (middle column) and 10.0 (right column) for $\alpha = 0°$.

Q4: How is vorticity generated?

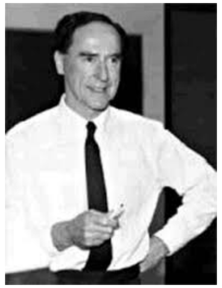
``Early'' treatments of the generation and decay of vorticity

LIGHTHILL - 1963

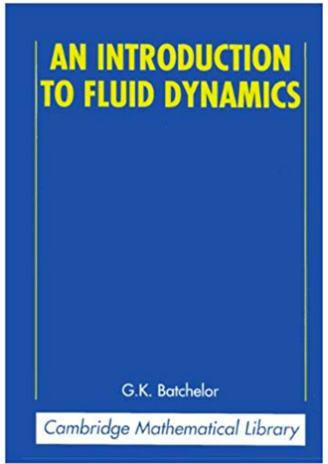
- Noted that at almost all points of the boundary there is a non-zero gradient of vorticity along the normal.
- That tangential vorticity must be created at the boundary, in the direction of the surface isobars and at a rate proportional to the tangential pressure gradient.
- Described the boundary as a distributed source of vorticity, similar to a distributed source of heat.

Lighthill, M.J. (1963) Introduction: Boundary Layer Theory: Laminar Boundary Layer. Oxford University Press, Oxford, 46113.

BATCHELOR - 1967



- Devoted a substantial section to the source of vorticity in motions started from rest.
- Identified the free-slip velocity in the inviscid solution for flow past a body set in motion as the effective source of vorticity in real flows.
- Argued that vorticity cannot be destroyed in the interior of a homogeneous fluid, and this appeared to lead to the concept of loss of vorticity by diffusion to boundaries.



3 important previous papers since that will be reference (there are others of course – e.g., Wu & Wu's papers)

Morton, B. R., 'The generation and decay of vorticity', Geophysical & Astrophysical Fluid Dynamics, 28:3, 277 - 308, 1984.

Rood, E. P., Myths, math, and physics of free-surface vorticity" in "Mechanics USA 1994" edited by AS Kobayashi, Appl Mech Rev vol 47, no 6, part 2, June 1994.

Lundgren, T. & Koumoutsakos, P., On the generation of vorticity at a free surface, J. Fluid Mech., 382, pp. 351-366, 1999.

Vorticity equation does not include generation – vorticity is generated only at boundaries/interfaces

The Helmholtz vorticity equation for an incompressible homogeneous fluid

$$\partial\boldsymbol{\omega}/\partial t + (\mathbf{v}\cdot\nabla)\boldsymbol{\omega} = (\boldsymbol{\omega}\cdot\nabla)\mathbf{v} + \nu\nabla^2\boldsymbol{\omega},$$

Processing term
describes the effects of local amplification (or concentration) of vorticity by vortex filament stretching and local turning of filaments

Viscous diffusion
spread of vorticity due to viscosity

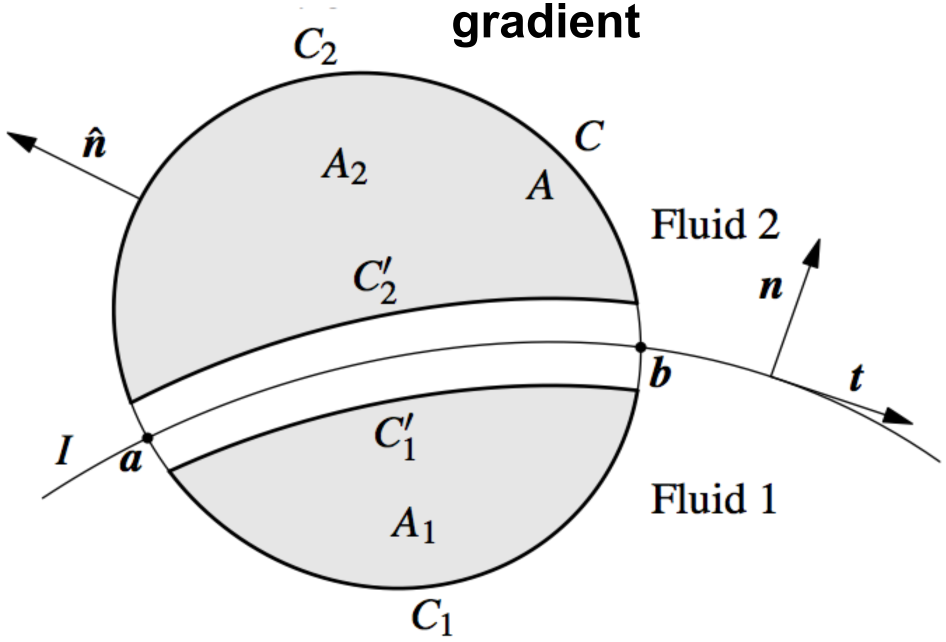
No true generation term
How is vorticity generated at boundaries?

Vorticity flux at an interface

$$[\![\sigma]\!] = \boxed{-\left[\!\left[\frac{\mathrm{d}\boldsymbol{u}}{\mathrm{d}t}\right]\!\right]\cdot\hat{\boldsymbol{t}} - \frac{\partial}{\partial s}\left[\!\left[\frac{p}{\rho}\right]\!\right]} - \frac{\partial}{\partial s}[\![\Phi_g]\!]$$

Acceleration **Tangential pressure gradient**

Φ_g is the body force potential, $\boldsymbol{g} = -\nabla\Phi_g$



$$\gamma = -[\![\boldsymbol{u}\cdot\boldsymbol{t}]\!]$$

Brøns et al., JFM, 2014.
Terrington, PhD, Monash University, 2019.

MORTON: Generation of vorticity at a solid boundary is an inviscid process

$$\sigma = \frac{d\boldsymbol{u}}{dt} \cdot \hat{\mathbf{t}} + \frac{\partial}{\partial x}\frac{p}{\rho} - \boldsymbol{g} \cdot \hat{\mathbf{t}}$$

Rate of vorticity generation = (Acceleration of lower boundary) – (Inviscid acceleration of fluid elements due to pressure/body forces)

Morton, B.R (1984) '*The generation and decay of vorticity*'

$$\delta k = \oint_{\mathscr{C}} \mathbf{v} \cdot d\mathbf{r} \approx (u - U)\delta x$$

Circulation: integral of velocity around a closed loop

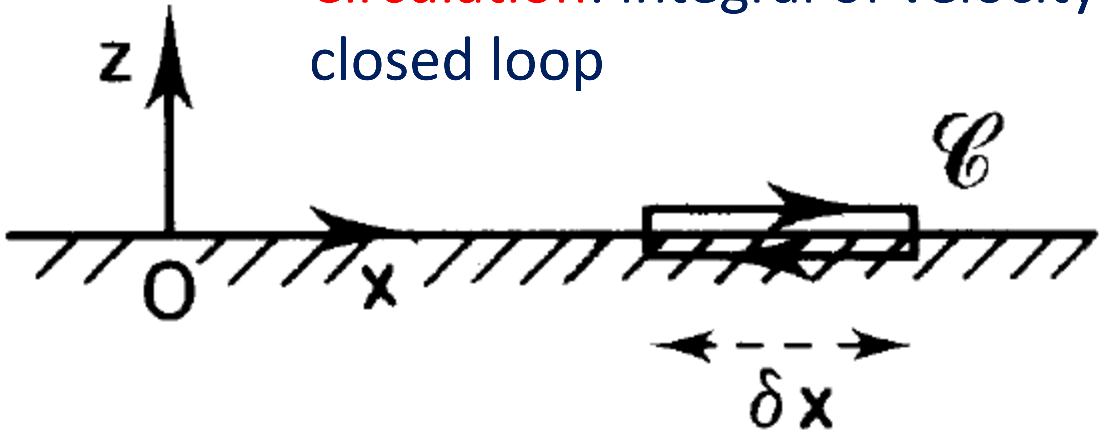


FIGURE 9 The circulation circuit 𝒞 for generation at a boundary.

Can our integration loop go into the solid?

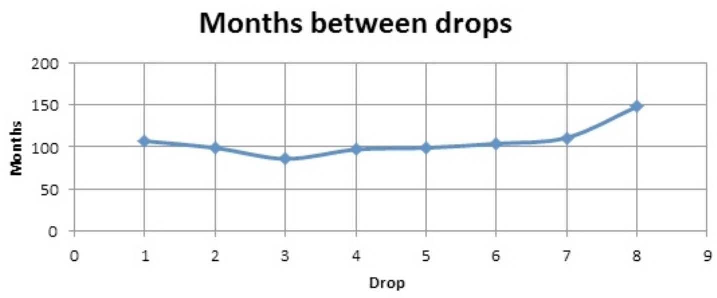
Can we have an inviscid fluid?

How can we regard a solid as a fluid?

"The most famous version of the experiment was started in 1927 by Professor Thomas Parnell of the University of Queensland in Brisbane, Australia, to demonstrate to students that some substances that appear to be solid are in fact very-high-viscosity fluids"[1].

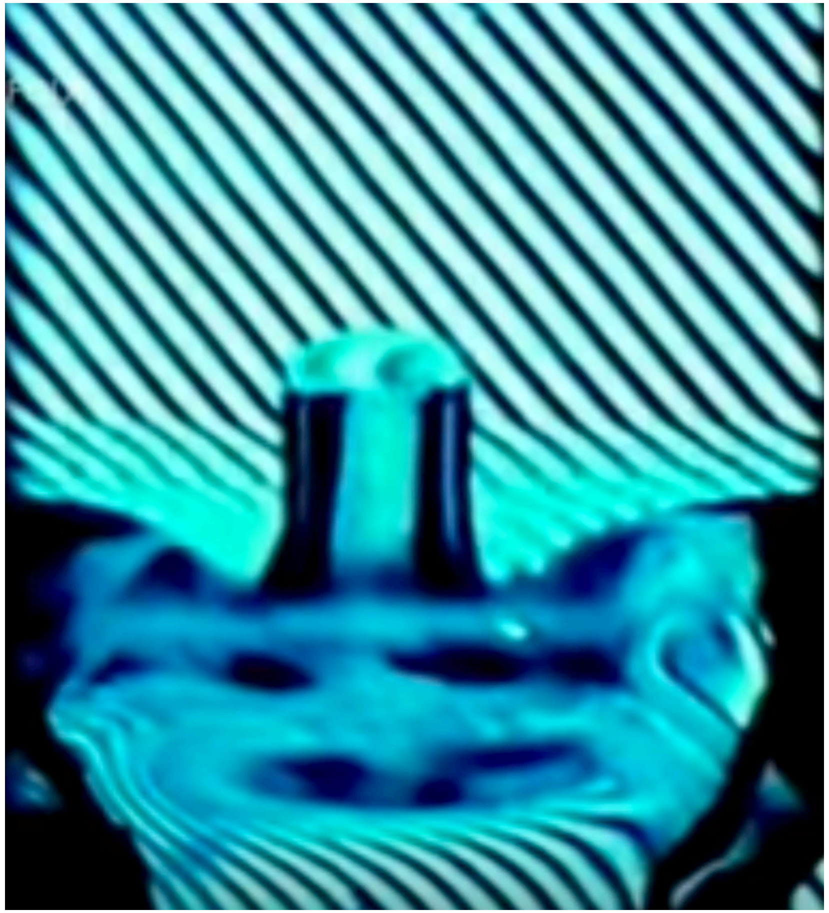
Professor John Mainstone (taken in 1990)

Pitch has a viscosity approximately 230 billion (2.3×10^{11}) times that of water



[1]*en.wikipedia.org/wiki/Pitch_drop_experiment*

Superfluids have zero viscosity



Superfluid fountain – frictionless flow forever

www.youtube.com/watch?v=2Z6UJbwxBZI

Morton (1984): Fluid pressure gradient and relative acceleration lead to vorticity production at boundaries/interfaces

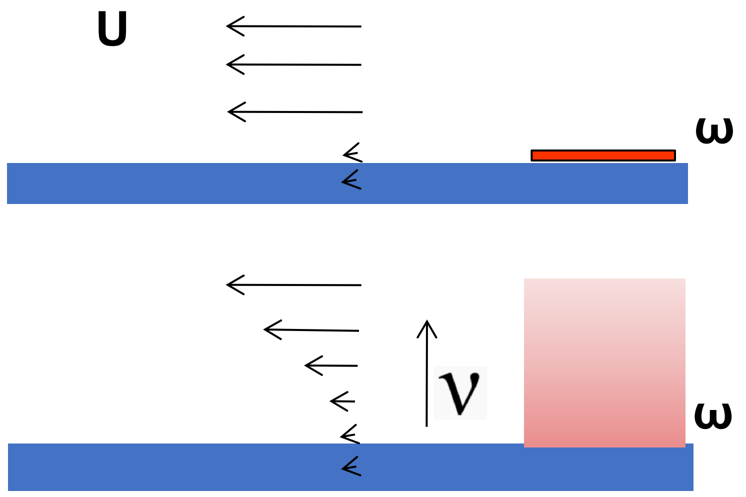
$$[\![\sigma]\!] = \frac{d\gamma}{dt} - \frac{\partial}{\partial s}\left[\!\!\left[\frac{p}{\rho}\right]\!\!\right]$$

- Vorticity is generated at boundaries by the relative acceleration of fluid and wall.
- In a homogeneous fluid, vorticity is generated only at boundaries and its generation is instantaneous and the result of inertial forces.

Wake vorticity generated at boundary by either: fluid pressure gradient

Fluid "acceleration"

t = 0



Instantaneous acceleration of fluid at t = 0

t > 0

Diffusion of vorticity due to viscosity

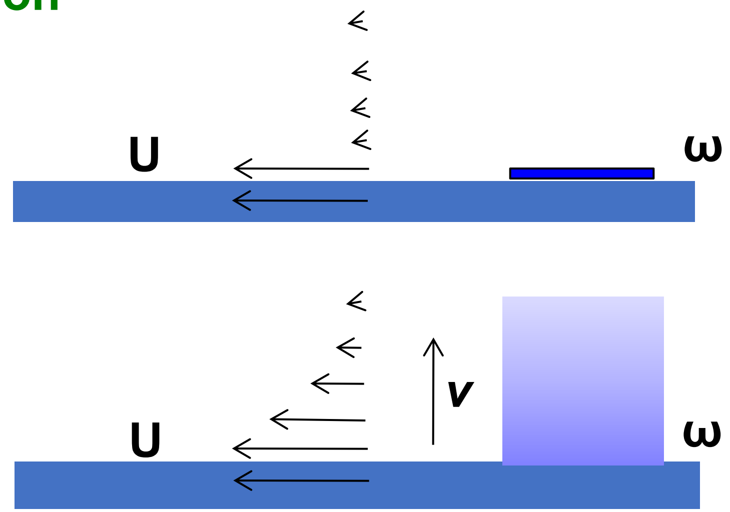
Note that generation is masked by viscous redistribution of the vorticity immediately upon generation.

... or
boundary acceleration

Boundary acceleration

t = 0

t > 0



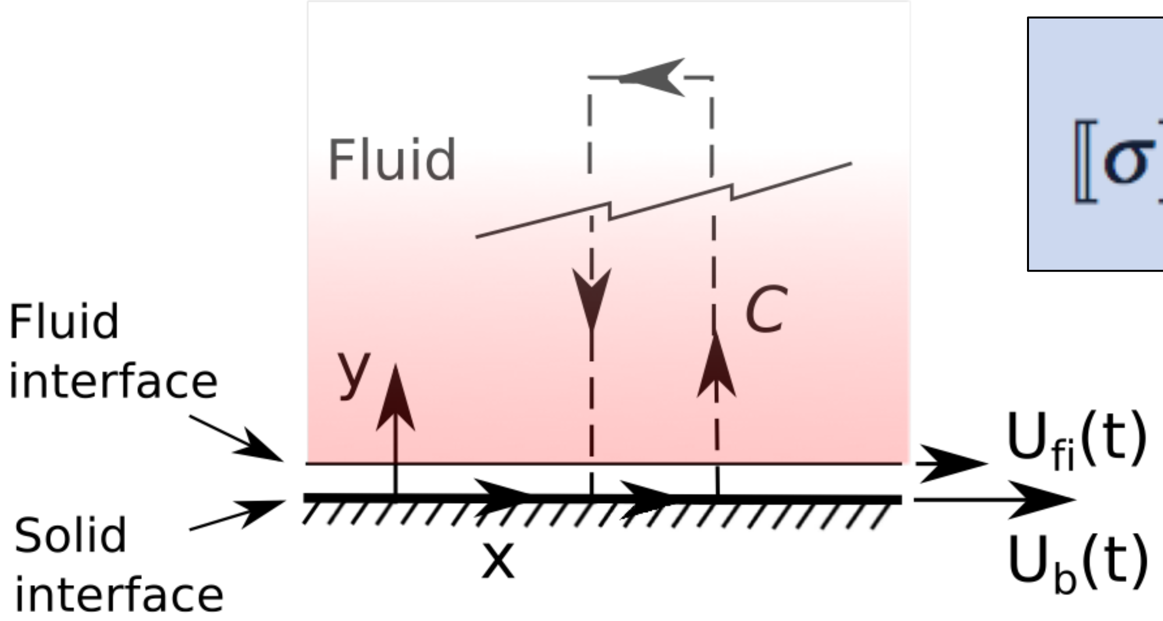
Instantaneous acceleration of surface at t = 0

Diffusion of vorticity due to viscosity

Vorticity generated due to a differential velocity being created by plate/fluid acceleration
No-slip condition? Infinitely thin layer of fluid accelerated with plate
What is the role of viscosity? Redistributes vorticity following generation

Q5: How is vorticity transported, and how does it decay?

Impulsively started plate



$$\boxed{[\![\sigma]\!] = \frac{d\gamma}{dt}} - \frac{\partial}{\partial s}\left[\!\left[\frac{p}{\rho}\right]\!\right]$$

Impulsive plate acc^n at t = 0

=0

$U(y,0) = U_b\, H(-y)$

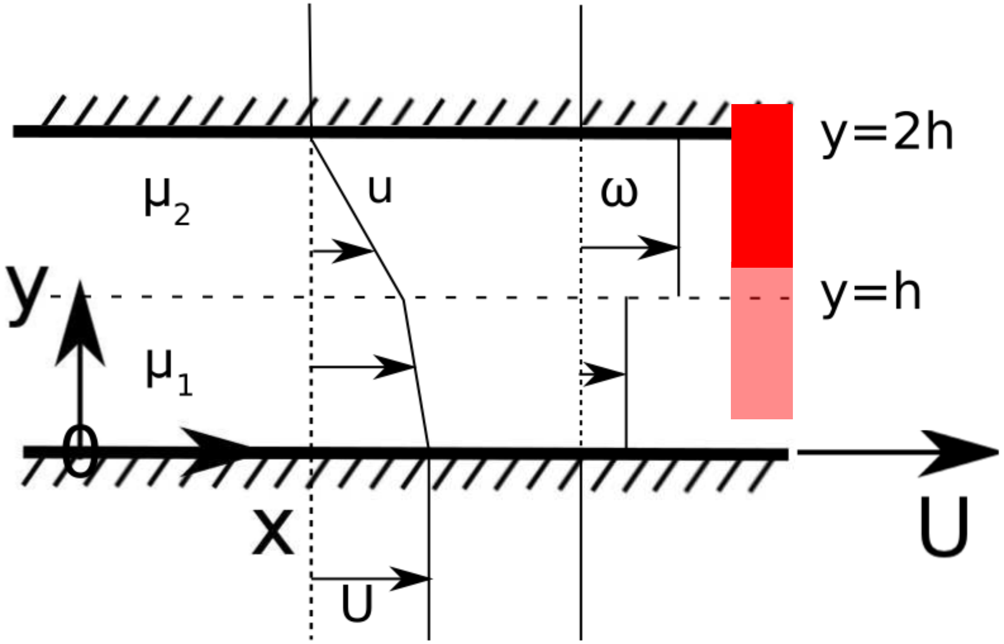
Vorticity/unit length = U_b

At t = 0, all the vorticity is generated.

t > 0, vorticity diffuses away from the boundary and remains most concentrated at boundary.

Vorticity is neither lost nor gained to solid boundary.

Plane Couette flow: two fluids



$$[\![\sigma]\!] = \frac{d\gamma}{dt} - \frac{\partial}{\partial s}\left[\!\left[\frac{p}{\rho}\right]\!\right]$$

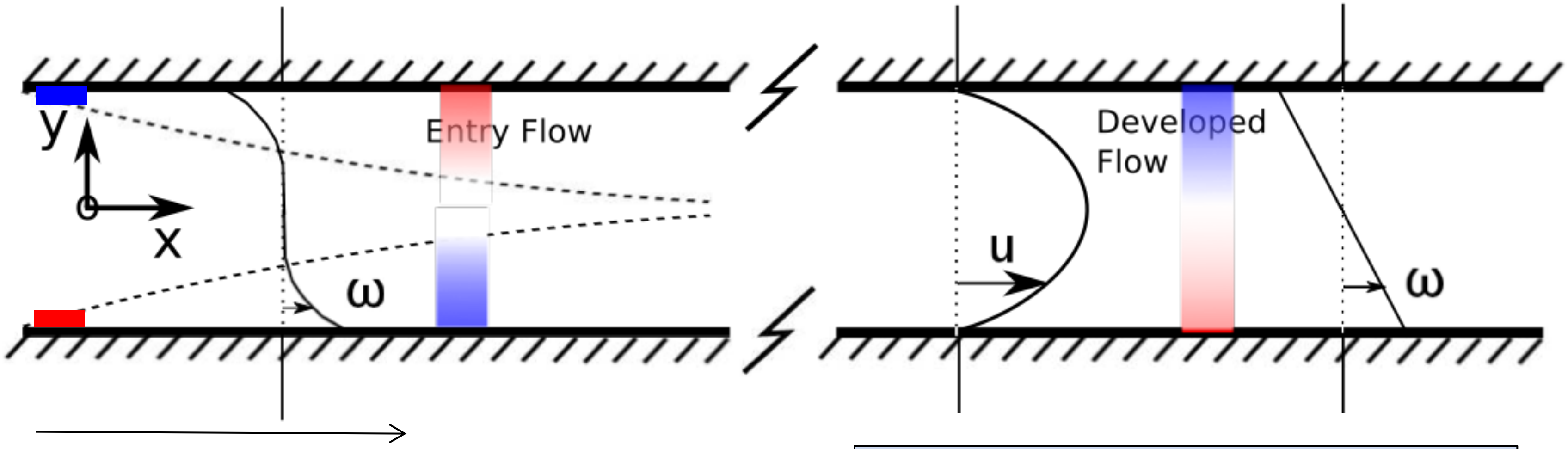
$$u_1 = (1 - \frac{\mu_2}{\mu_1+\mu_2}(\frac{y}{h}))U, \qquad u_2 = -\frac{2\mu_1}{\mu_1+\mu_2}(\frac{1}{2h})U$$

$$\omega_1 = \frac{\mu_2}{\mu_1+\mu_2}(\frac{U}{h}), \qquad \omega_2 = \frac{\mu_1}{\mu_1+\mu_2}(\frac{U}{h})$$

Circulation/unit length = U

- **An amount of vorticity U per unit length of plate is generated at t=0.**
- **This diffuses out into the layer to produce in time two layers, each with uniform vorticity distribution.**
- **Thereafter the flow is steady and vorticity is neither generated at the plates nor lost to them.**
- **Due to different μ, at the centre interface, there is a vorticity jump but velocity is continuous (as at other interfaces).**

Poiseuille flow – no loss of vorticity at boundaries



Axial pressure gradient:

$$[\![\sigma]\!] = \frac{d\gamma}{dt} - \frac{\partial}{\partial s}\left[\!\left[\frac{p}{\rho}\right]\!\right]$$

- Vorticity is generated continuously by the tangential pressure gradient at the lower boundary, *and* at the upper boundary at same rate *(sense of normal reversed)*
- Each diffuses towards the centre plane where the positive and negative fluxes suffer annihilation.
- The circulation per unit length of channel is zero.

The decay of vorticity

- Vorticity is not lost by diffusion to boundaries other than in circumstances in which vorticity of counter sign is being generated and suffering immediate cross-diffusive annihilation with pre-existing vorticity.

- The gross circulation in an enveloping contour of a bounded region with outer boundary at rest must always be zero.

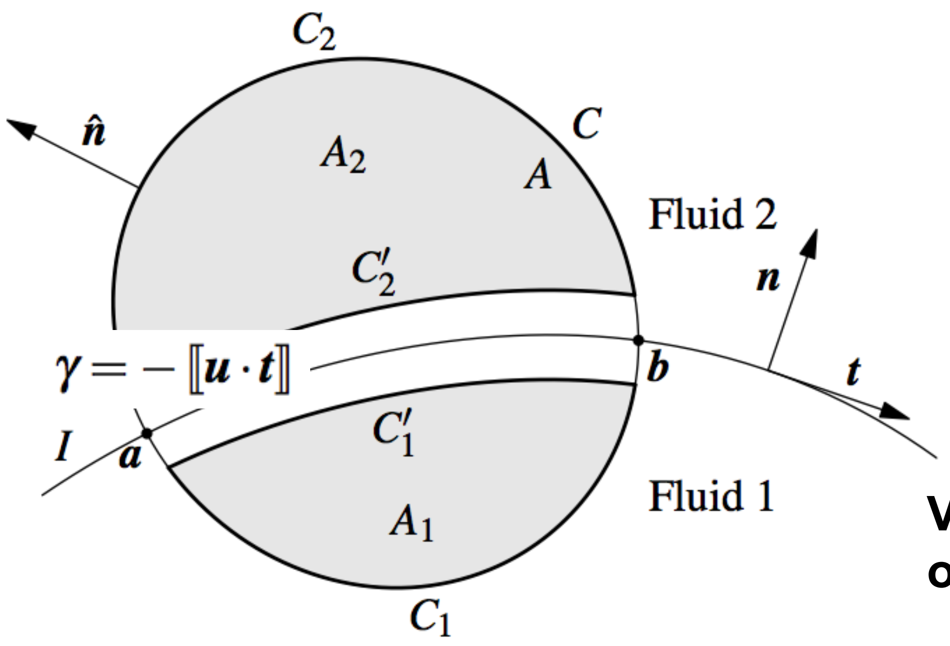
- Diffusion will, therefore, always lead in time to the total annihilation of all vorticity, through the diffusion of momentum and the corresponding wall stresses.

Q6: Is vorticity conserved?

- Conservation laws play an important role in our understanding of fluid mechanics, and indeed our understanding of the universe. For example, the fundamental equations of fluid mechanics (continuity, Navier-Stokes and energy) equations are usually interpreted as expressions of the conservation of mass, momentum, and energy, respectively.
- Conservation laws often allow the analysis of a system in terms of inlet and outlet properties, as conserved quantities cannot be created or destroyed in the system interior.

Conservation of vorticity

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_A \omega \,\mathrm{d}A + \int_a^b \gamma \,\mathrm{d}s\right) = \oint_C \nu \nabla\omega \cdot \hat{\boldsymbol{n}} \,\mathrm{d}s - \left(\left[\!\left[\frac{p}{\rho}\right]\!\right]_b - \left[\!\left[\frac{p}{\rho}\right]\!\right]_a\right)$$



For no net pressure gradient, flux at outer boundaries:

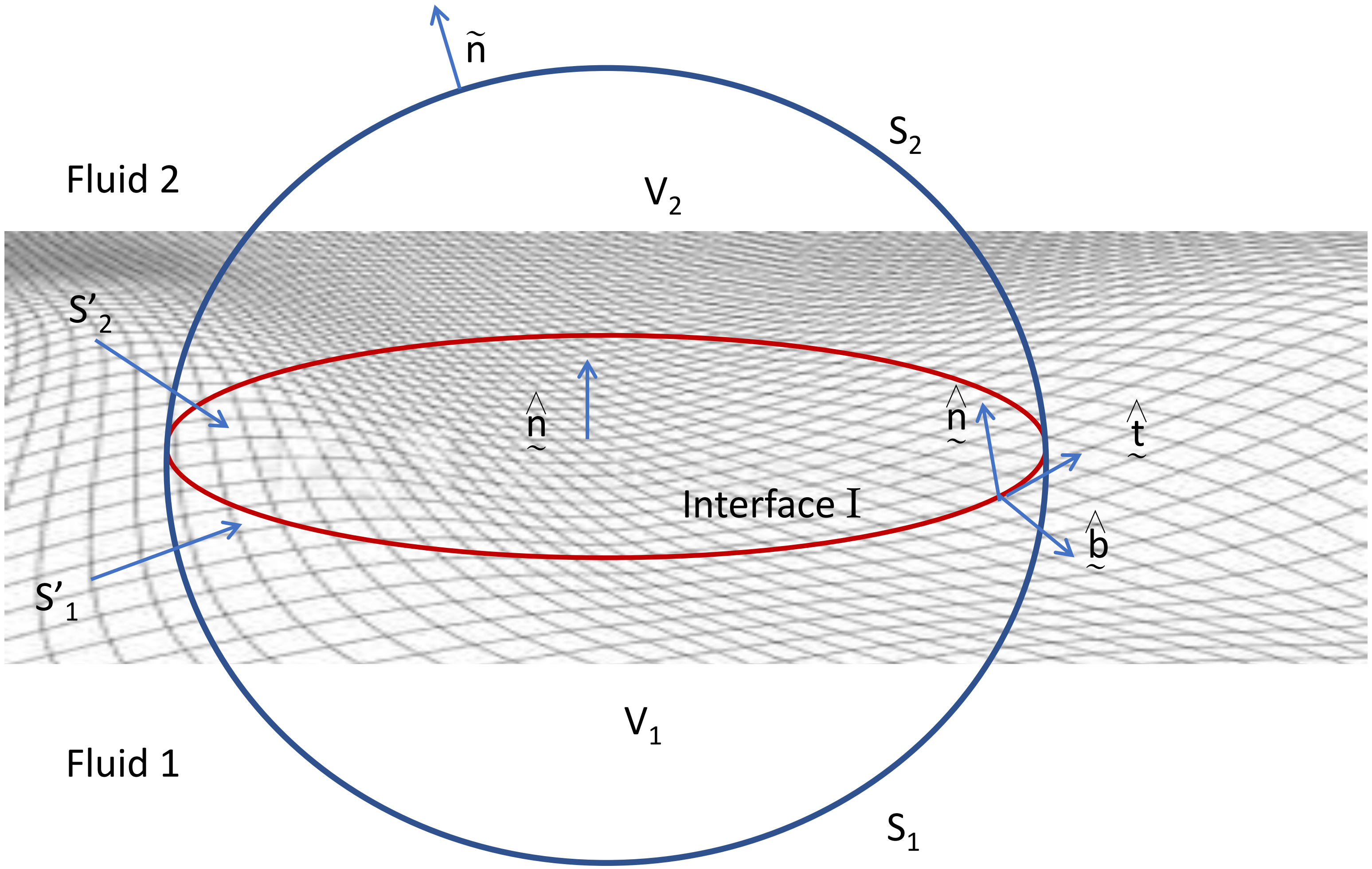
$$\boxed{\int_A \omega \,\mathrm{d}A + \int_a^b \gamma \,\mathrm{d}s} = \text{constant.}$$

Vorticity in body of fluid

Vorticity in vortex sheet at interface

Vorticity is conserved – only need to know what happens at the boundary to know how much vorticity is inside

- Vorticity conservation appears to be a global conservation property, under appropriate conditions, with vorticity generation occurring on a local scale.
- Vorticity generation of one sign must be balanced by creation of an equal magnitude of opposite sign circulation elsewhere in the domain.
- Transient and steady-state behaviour in many flows can then be understood in terms of this conservation principle: vorticity may be generated on a boundary, however the total amount of vorticity generated on all boundaries is often zero.
- The only means by which vorticity generated on a boundary may disappear from the flow is by annihilation due to cross-diffusion with opposite sign vorticity.



$\tilde{n}$
S_2
Fluid 2
V_2
S'_2
$\hat{\underset{\sim}{n}}$
$\hat{\underset{\sim}{n}}$
$\hat{\underset{\sim}{t}}$
Interface I
$\hat{\underset{\sim}{b}}$
S'_1
V_1
Fluid 1
S_1

What happens in 3D with an interface between 2 fluids?

Rate of change of total vorticity in control volume V with outer control volume

$\partial V = S_1 \cup S_2$

$$\frac{\mathrm{d}\boldsymbol{\Omega}}{\mathrm{d}t} = \oint_{\partial V} \boldsymbol{\omega}(\boldsymbol{v}^{\mathrm{b}} - \boldsymbol{u}) \cdot \boldsymbol{n}\mathrm{d}S + \oint_{\partial V} (\boldsymbol{\omega} \cdot \boldsymbol{n})\boldsymbol{u}\mathrm{d}S + \oint_{\partial V} \boldsymbol{\sigma}_b \mathrm{d}S - \oint_{\partial I} \left[\!\left[\frac{p}{\rho} + \Phi_g \right]\!\right] \mathrm{d}\boldsymbol{s}$$
$$- \oint_{\partial I} \frac{1}{2} [\![\boldsymbol{u} \cdot \boldsymbol{u}]\!] \mathrm{d}\boldsymbol{s} - \oint_{\partial I} \gamma(\boldsymbol{v} \cdot \hat{\boldsymbol{b}}) \mathrm{d}\boldsymbol{s} - \oint_{\partial I} (\gamma \cdot \hat{\boldsymbol{b}})(\boldsymbol{u} \cdot \hat{\boldsymbol{n}})\hat{\boldsymbol{n}} \mathrm{d}s,$$

The only source term is the pressure acceleration, which represents the generation of interface circulation due to the inviscid relative acceleration of fluid elements on each side of the interface (extension of Morton's description in 2D).

Viscous tangential acceleration and vortex stretching produces a change in interface circulation, but these are balanced by a decrease of vorticity in the fluid interior.

The 9 Commandments of Morton for flow with a solid wall boundary

1. that generation results from tangential acceleration of a boundary, from tangential initiation of boundary motion and from tangential pressure gradients acting along a boundary;
2. that generation is instantaneous;
3. that vorticity once generated cannot subsequently be lost by diffusion to boundaries;
4. that reversal of the sense of acceleration or of the sense of pressure gradient results in reversal of the sense of vorticity generated;
5. that wall stress relates to the presence of vorticity but is not a cause of its generation;
6. that the generation process is independent of the prior presence of vorticity;
7. that both senses of vorticity are needed to explain observations;
8. that walls play no direct role in the decay or loss of vorticity;
9. that vorticity decay results from cross-diffusion of two fluxes of opposite sense and takes place in the fluid interior.

Need a tenth Commandment for a free surface boundary

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