

Ph.D. Course on *Vorticity, Vortical Flows and Vortex-Induced Vibrations*  
Technical University of Denmark, Copenhagen, Denmark  
[vortex.compute.dtu.dk](http://vortex.compute.dtu.dk)  
August 26-30, 2019

# Bluff Body Wakes III: Effect of body rotation

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# Reading Material

## Isolated Rotating Cylinder

Akoury, R.E., Braza, M., Perrin, R., Harran, G., Hoarau, Y., 2008. The three-dimensional transition in the flow around a rotating cylinder. *Journal of Fluid Mechanics* 607, 1–11.

Mittal, S., Kumar, B., 2003. Flow past a rotating cylinder. *Journal of Fluid Mechanics* 476, 303–334.

Prandtl, L., 1926. Application of the “Magnus Effect” to the Wind Propulsion of Ships. Technical memorandum, National Advisory Committee for Aeronautics.

Rao, A., Leontini, J., Thompson, M.C., Sheridan, J. & Hourigan, K., A review of rotating cylinder wake transitions, *Journal of Fluid and Structures*, 53, 2 - 14, 2015.

Rao, A., Leontini, J., Thompson, M.C. & Hourigan, K., Three-dimensionality in the wake of a rotating cylinder in a uniform flow, *Journal of Fluid Mechanics*, 717, 1-29, 2013.

Rao, A., Leontini, J., Thompson, M.C. & Hourigan, K., Three-dimensionality in the wake of a rapidly rotating cylinder in uniform flow, *Journal of Fluid Mechanics*, 730, 379-391, 2013.

Radi, A., Thompson, M.C., Rao, A., Hourigan, K. & Sheridan, J., Experimental evidence of new three-dimensional modes in the wake of a rotating cylinder, *Journal of Fluid Mechanics*, 734, 567-594, 2013.



## Isolated Rotating Sphere

Giacobello, M., Ooi, A., & Balachandar, S., Wake structure of a transversely rotating sphere at moderate Reynolds numbers. *Journal of Fluid Mechanics*, 621, 103-130, 2009.

Poon, E., Ooi, A., Giacobello, M., Iaccarino, G., & Chung, D., Flow past a transversely rotating sphere at Reynolds numbers above the laminar regime. *Journal of Fluid Mechanics*, 759, 751-781, 2014.

Williamson, C.H.K. & Govardhan, R., 2004 Vortex-induced vibrations. *Annu. Rev. Fluid Mech.* 36 (1), 413–455.

# Lecture Objectives

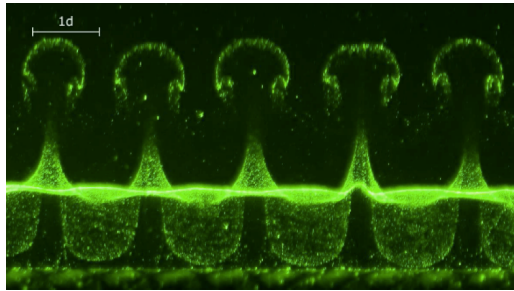
## Learn:

**Applications of rotating bodies**

**Magnus effect**

**Effect of body rotation on wakes of, and forces on, cylinders and spheres**

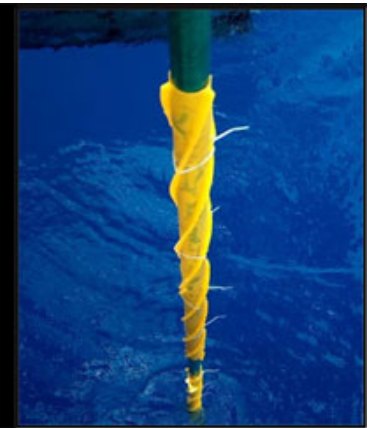
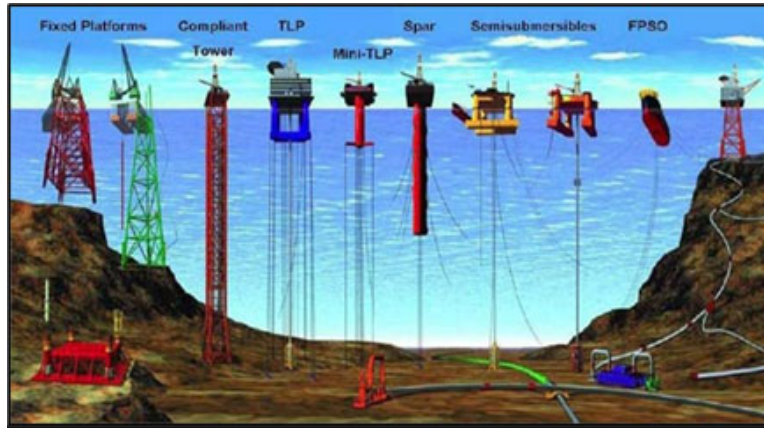
# Outline of Presentation



- Motivation
- Non-rotating cylinder/sphere, cross-flow: 2d/axisymmetric & 3d modes
- Rotating cylinder, no cross-flow
- Rotating cylinder/sphere in cross-flow: Varying rotation rate and Reynolds number
- Applications: Flettner rotors, ball sports, wind turbines
- Conclusions

# Many applications for FSI and *non-rotating* bodies

- Offshore marine industry
- Wind engineering
- Heat exchangers
- Aerospace
- Tethered bodies
- etc



Ocean engineering applications, such as marine risers, oil platforms and mooring lines

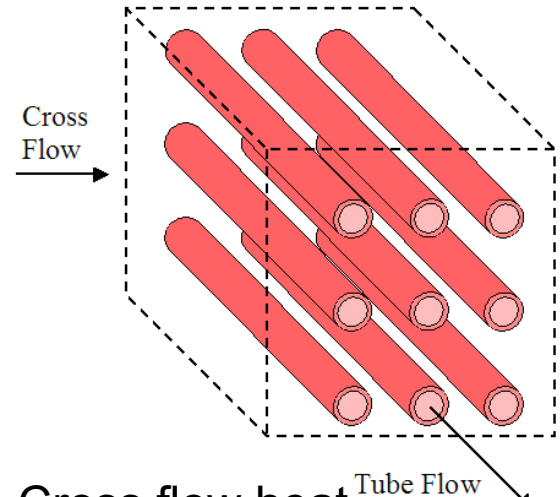
[web.mit.edu/towtank/www/viv.html](http://web.mit.edu/towtank/www/viv.html)



Tall buildings

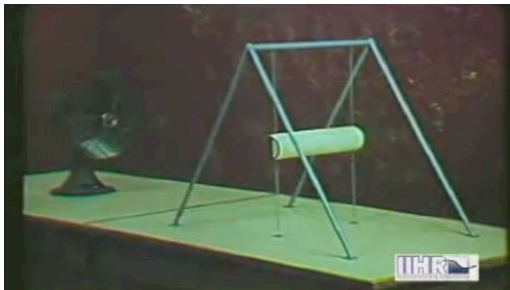
(Imperial Tower – Mumbai)

<http://gizmodo.com/how-this-116-story-skyscraper-will-confuse-the-wind-508206826>



Cross flow heat exchangers

[www.real-world-physics-problems.com/heat-exchanger.html](http://www.real-world-physics-problems.com/heat-exchanger.html)



# ..and for *rotating* bodies

- Flettner rotors
- Wind turbines
- Sports balls
- Rolling particles
- etc



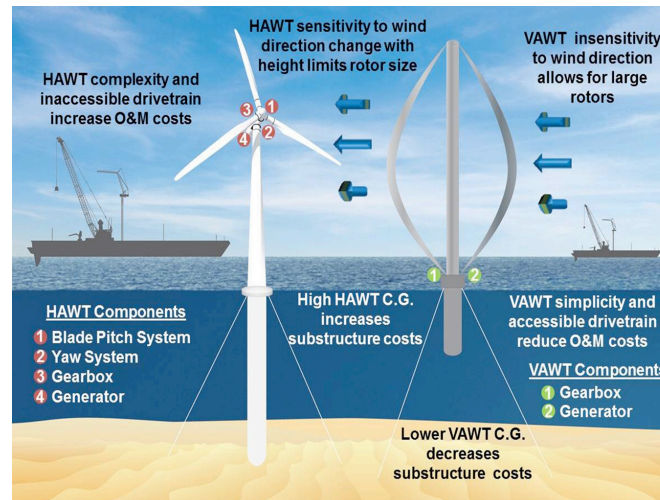
Flettner Rotor: Buckau (1924)

[en.wikipedia.org/wiki/Rotor\\_ship](http://en.wikipedia.org/wiki/Rotor_ship)



Flettner wings

[www.youtube.com/watch?v=hlmvHfIASzo](http://www.youtube.com/watch?v=hlmvHfIASzo)

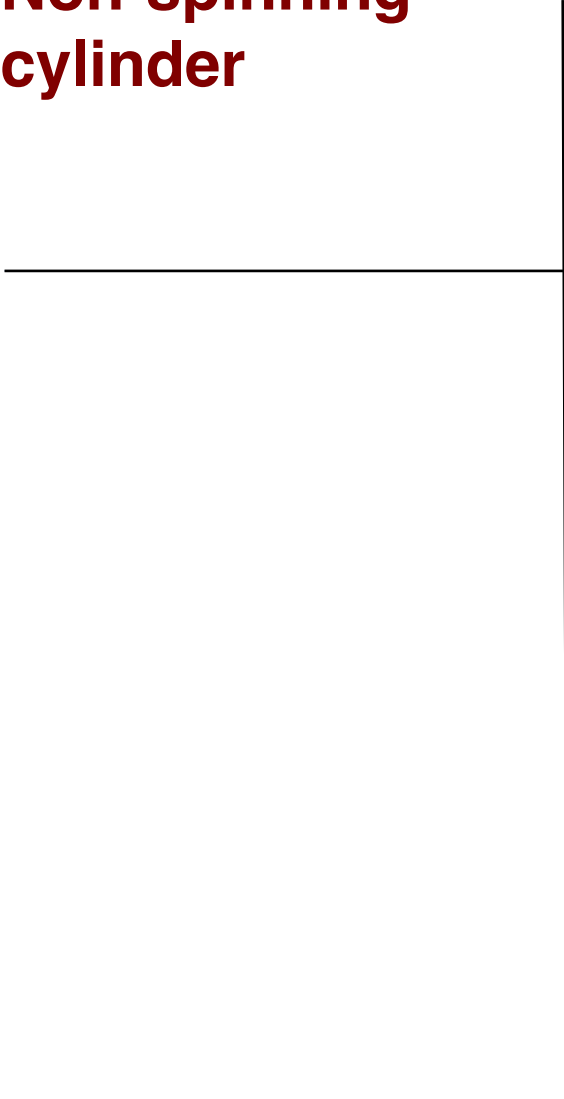


Vertical axis wind turbines

[www.offshorewind.biz/2012/07/31/usa-sandia-re-evaluates-vertical-axis-wind-turbines/](http://www.offshorewind.biz/2012/07/31/usa-sandia-re-evaluates-vertical-axis-wind-turbines/)

[commons.wikimedia.org/wiki/File:Vertical\\_wind\\_turbine\\_near\\_US\\_Capitol.jpg](https://commons.wikimedia.org/wiki/File:Vertical_wind_turbine_near_US_Capitol.jpg)

**Non-spinning  
cylinder**



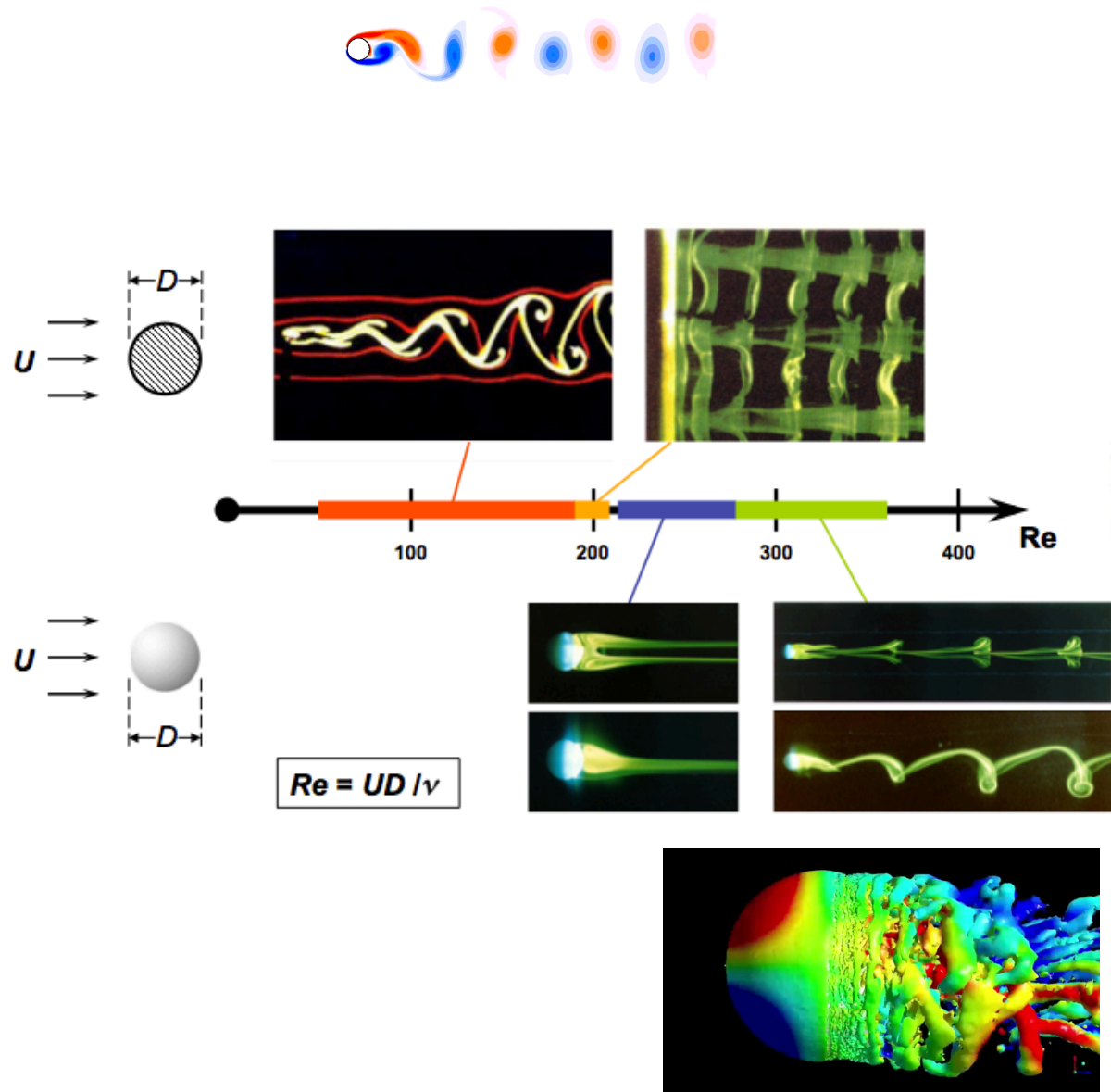
# Isolated cylinder and sphere have important wake transitions at moderate Re

## Cylinder

- Unsteady 2s at  $Re = 47$
- 3d mode A at  $Re = 180$
- 3d mode B at  $Re = 250$

## Sphere

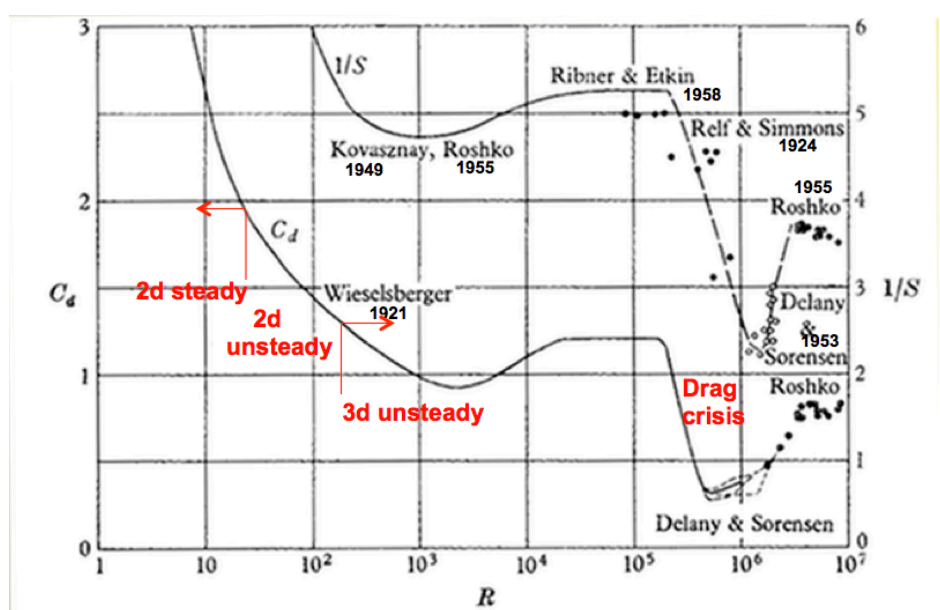
- Steady asymmetrical at  $Re = 212$
- Unsteady symmetrical at  $Re = 272$  (Hopf)



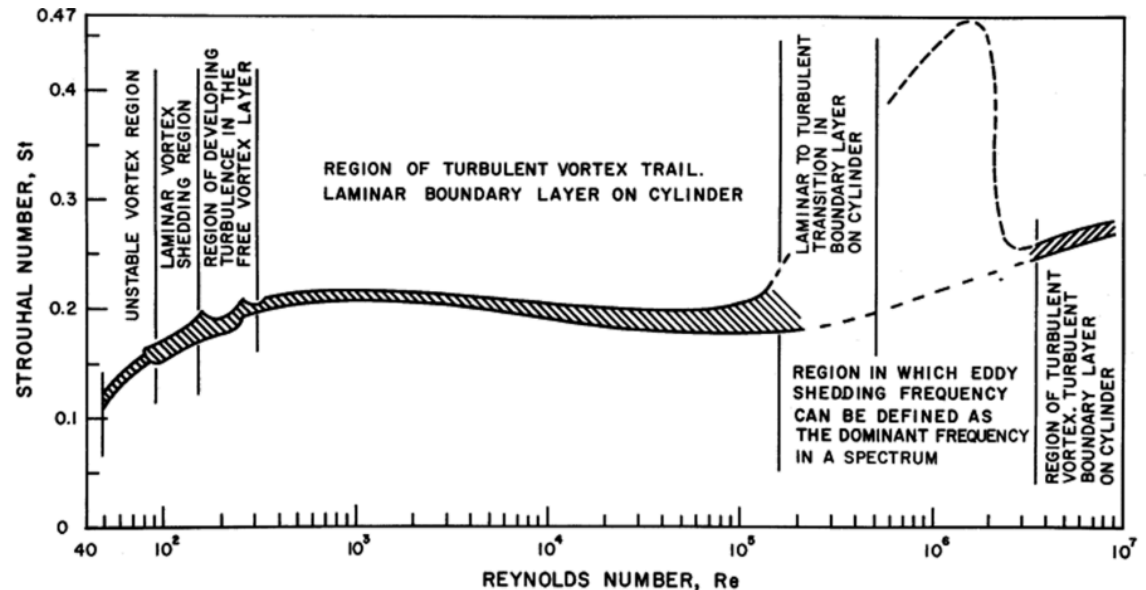


# Cylinder wake transitions

- Strouhal number and drag coefficient variation
- Initial transitions in low-moderate Re range



Drag coefficient and reciprocal of Strouhal number.  
Roshko (JFM, 1961)

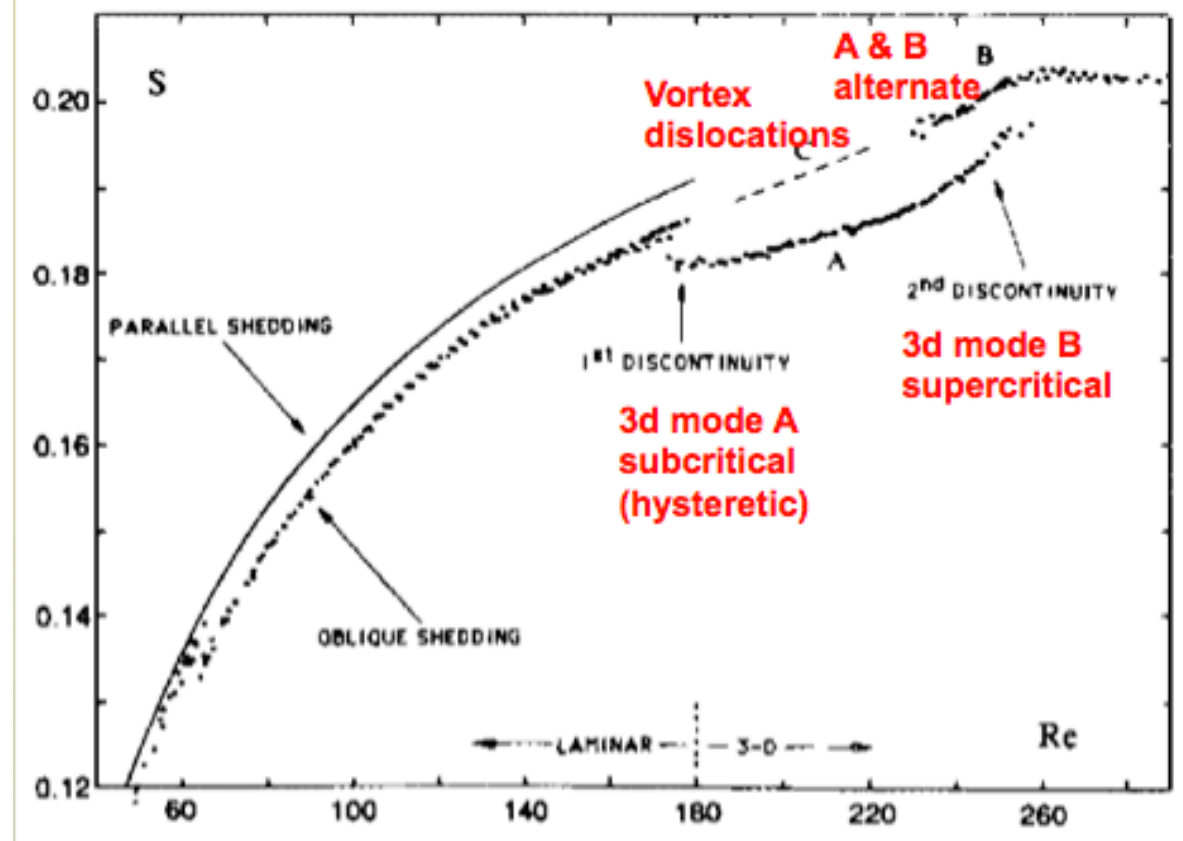


Strouhal number vs Reynolds number



# Cylinder wake transitions

- Strouhal number and drag coefficient variation
- Low Reynolds number transitions

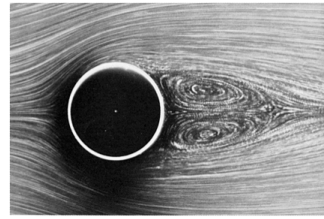


Strouhal number vs Reynolds number

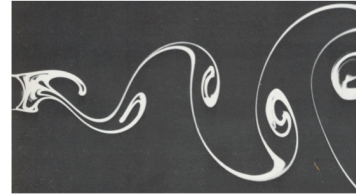
Williamson (PoF, 1988)

# Cylinder: Major wake transitions as Reynolds number increases

Steady wake behind a circular cylinder at  $Re = 26$   
Photograph: S. Taneda (Van Dyke 1982)



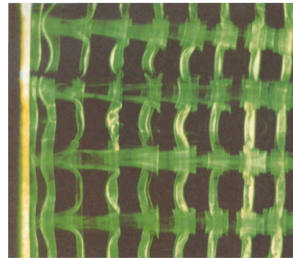
2D vortex street behind a circular cylinder at  $Re = 140$   
Photograph: S. Taneda (Van Dyke 1982)



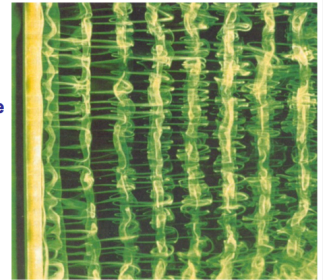
$0 < Re < 47$ :  
Steady 2D wake

$47 < Re < 180$ :  
Periodic 2D vortex street

Mode A instability in the wake behind a circular cylinder at  $Re = 200$   
Williamson (1996)



Mode B instability in the wake behind a circular cylinder at  $Re = 270$   
Williamson (1996)



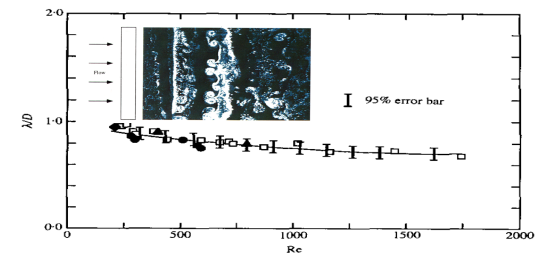
$Re = 190$ : Subcritical  
Mode A instability  
( $\lambda_d \approx 4d$ )

$Re = 240$ : Mode B  
instability ( $\lambda_d \approx 1d$ )

Turbulent wake behind a circular cylinder at  $Re = 2000$   
Photograph: ONERA, Werlé & Gallon - 1972 (Van Dyke 1982)



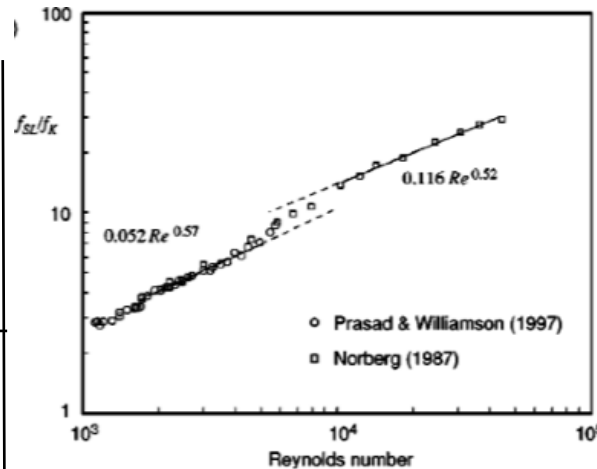
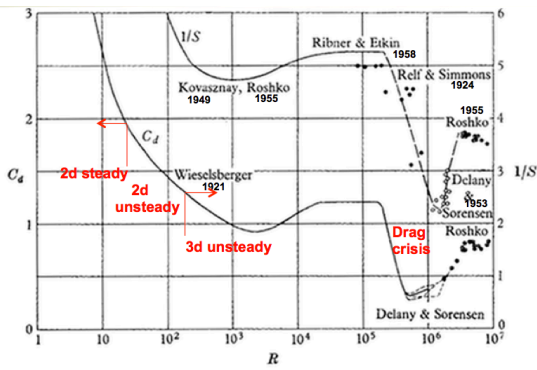
Re increasing: spatio-temporal chaos, rapid transition to turbulence



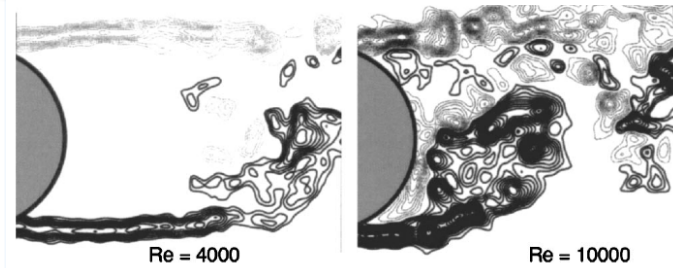
Persistence of mode B

# Other cylinder wake transitions

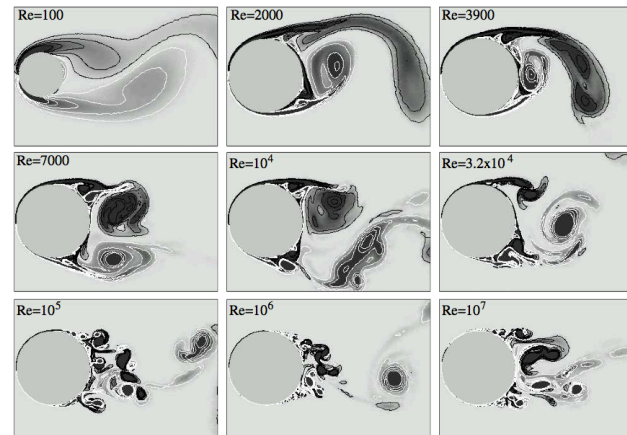
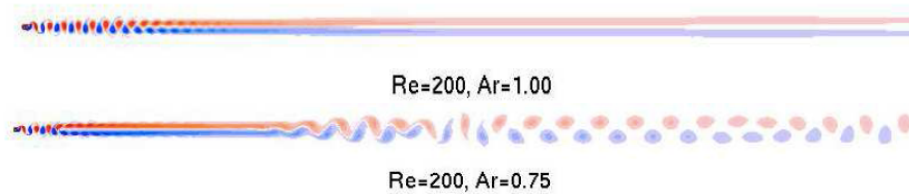
- Shear layer instability  
 $Re > 10^2$
- Far wake instability –  
wake relaminarises into  
double shear layer
- Drag crisis at  $Re \approx 2 \times 10^5$



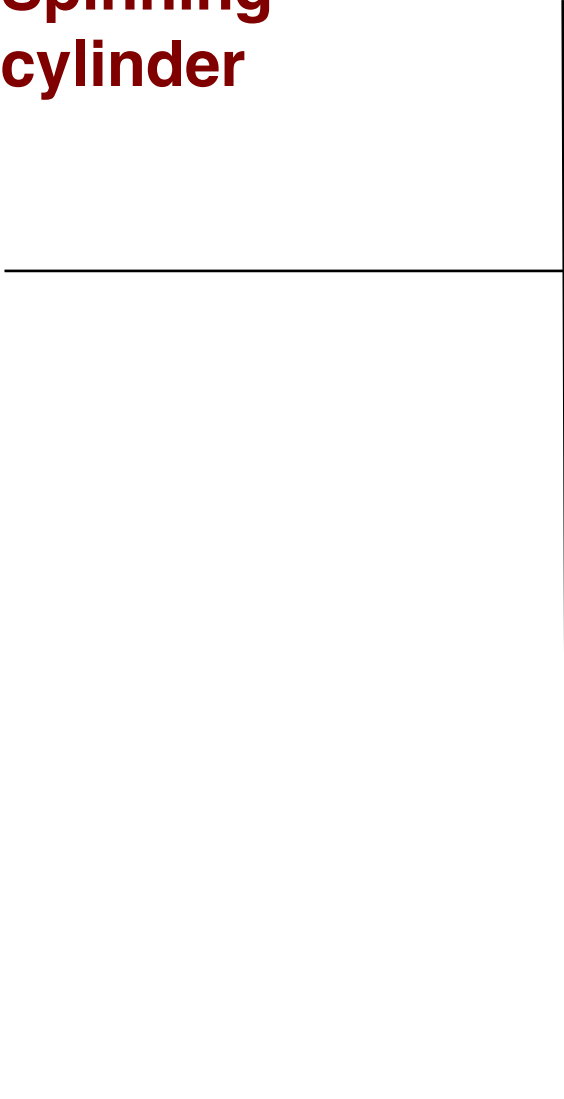
Thompson & Hourigan, *PoF*, 2005



N. Saelim and D. Rockwell, *private comm.* (2004)

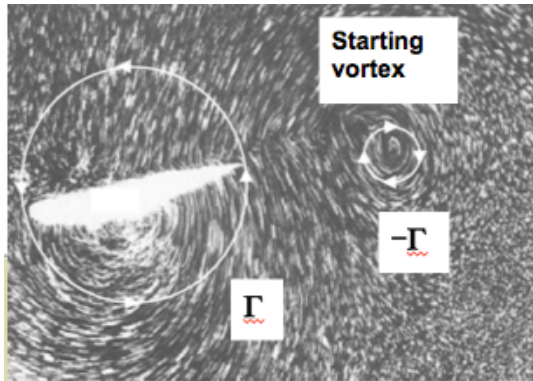


**Spinning  
cylinder**

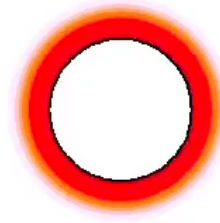


# Spinning cylinder generates net vorticity, starting vortex

- Spin a cylinder and vorticity generated diffuses to infinity
- Add a uniform flow and a starting vortex is formed
- Net lift results, similar to an airfoil

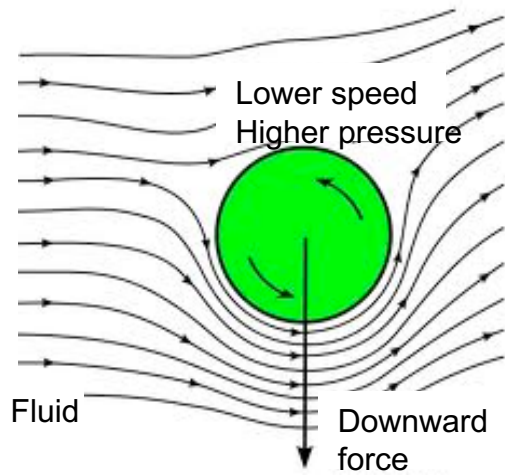


Impulsively spin a cylinder in still fluid



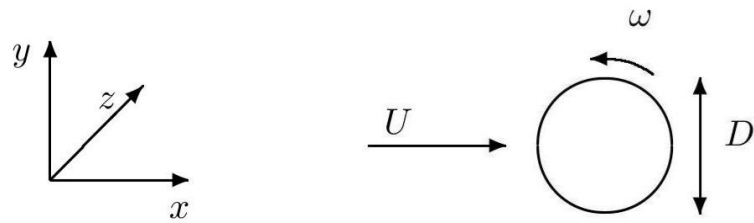
Add a cross flow and spin the cylinder

# Magnus Effect – provides lift



[www.aviation-for-kids.com/the-magnus-force.html](http://www.aviation-for-kids.com/the-magnus-force.html)

# CFD Methods



Reynolds number  $Re = UD/\nu$

Rotation rate  $\alpha = \omega D/2U$

## Spectral Element Method

The incompressible Navier-Stokes (N-S) equations are solved in 2-d and 3-d.

An in-house numerical solver employing a spectral element technique was used.

The computational domain consisted of quadrilateral macroelements with internal node points.

Fractional time-stepping technique was used to integrate the pressure, advection and diffusion terms of the N-S equation.

## Stability analysis

The Floquet multiplier ( $\mu$ )

$|\mu| < 1$ , stable

$|\mu| = 1$ , neutrally stable

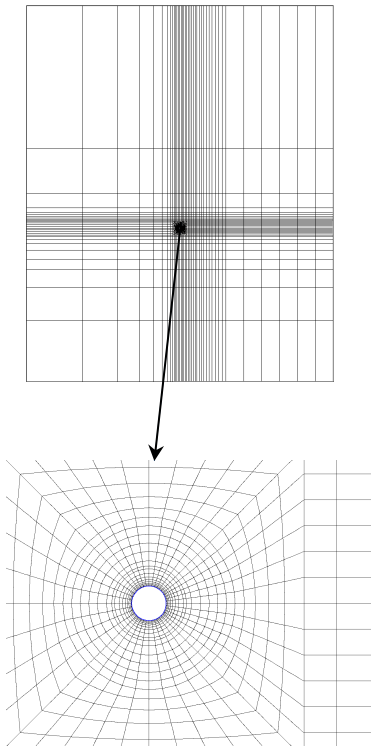
$|\mu| > 1$ , unstable

$\mu$  real & positive: 3d mode and base flow synchronous (A & B)

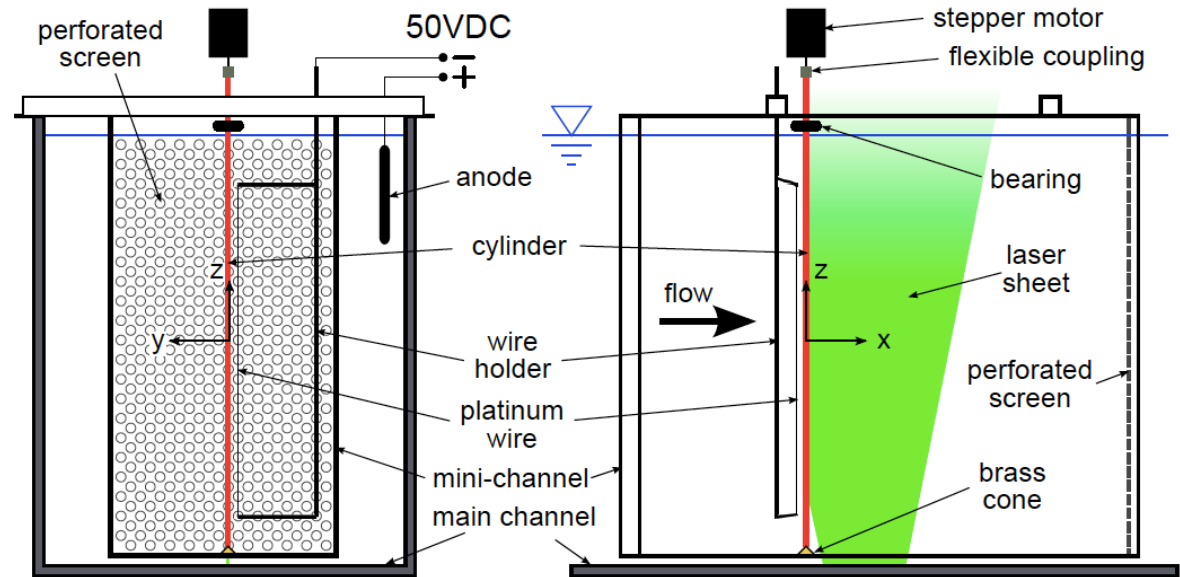
$\mu$  complex: if base flow periodic, 2<sup>nd</sup> freq  $\Rightarrow$  quasi-periodic 3d flow

if base flow steady, 3d flow is periodic

$\mu$  real & negative: subharmonic mode



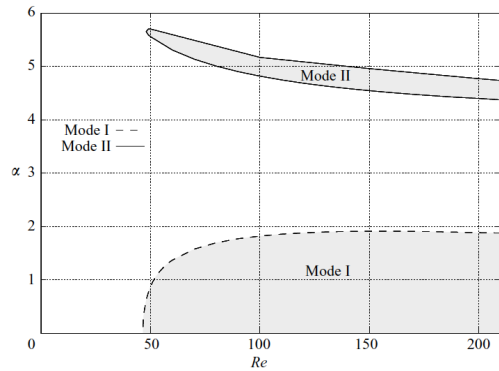
# Water Channel



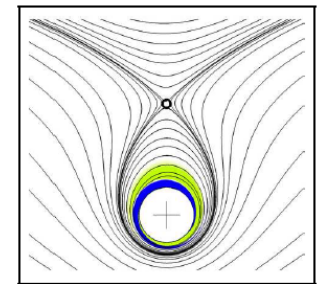
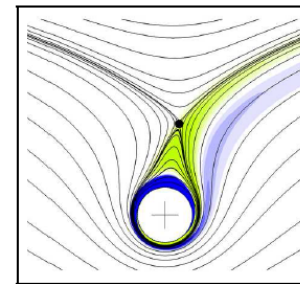
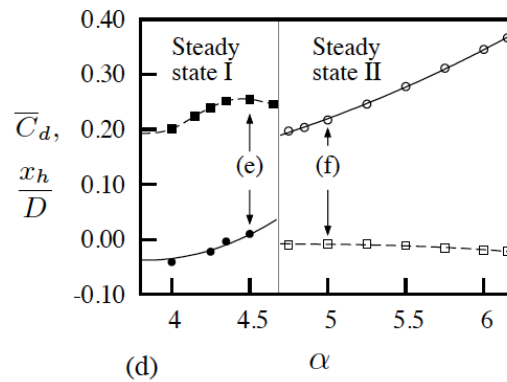
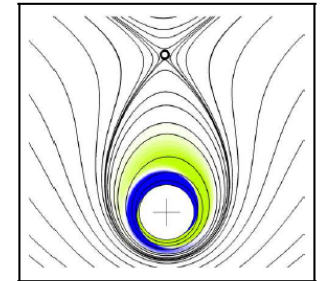
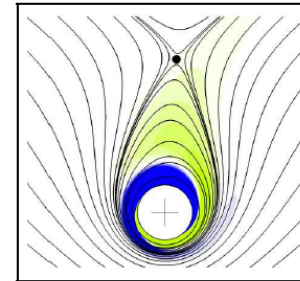
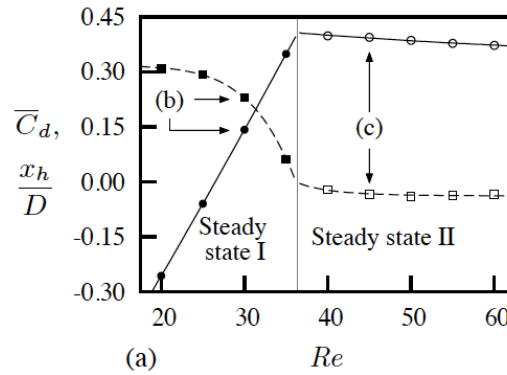
Water channel set up (FLAIR)



# Rotating Cylinder – 2 Steady States

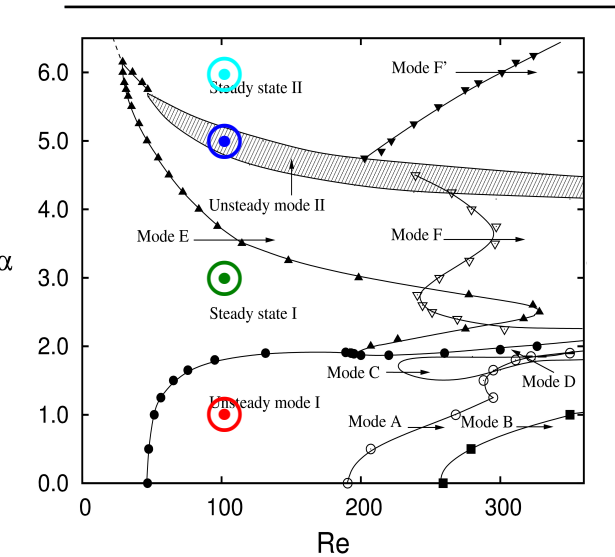


$\alpha = 6.15$



$Re = 200$

# Rotate Cylinder & Add Flow 2d States



⊙  $Re = 100, \alpha = 1$



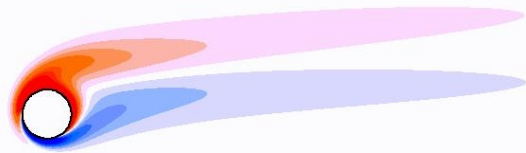
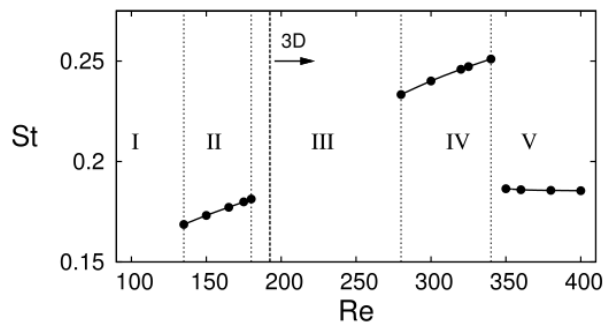
⊙  $Re = 100, \alpha = 3$



⊙  $Re = 100, \alpha = 5$



⊙  $Re = 100, \alpha = 6$



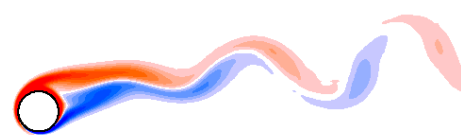
$Re=50$



$Re=250$



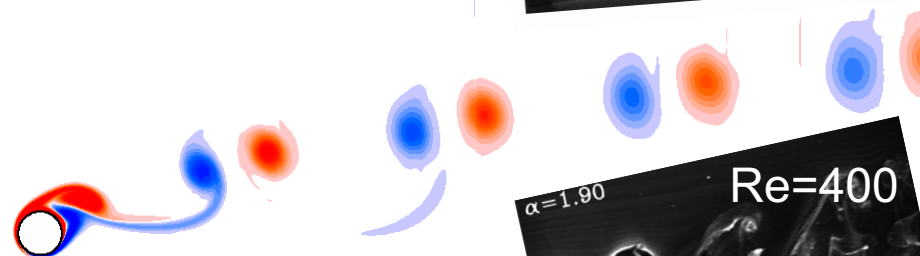
$Re=100$



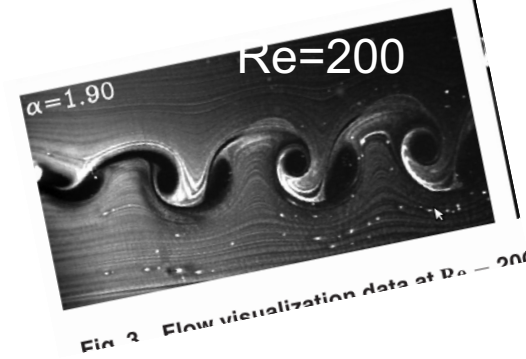
$Re=300$



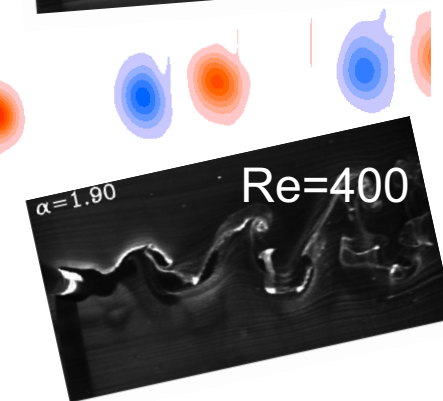
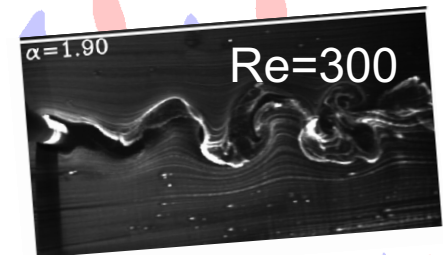
$Re=180$



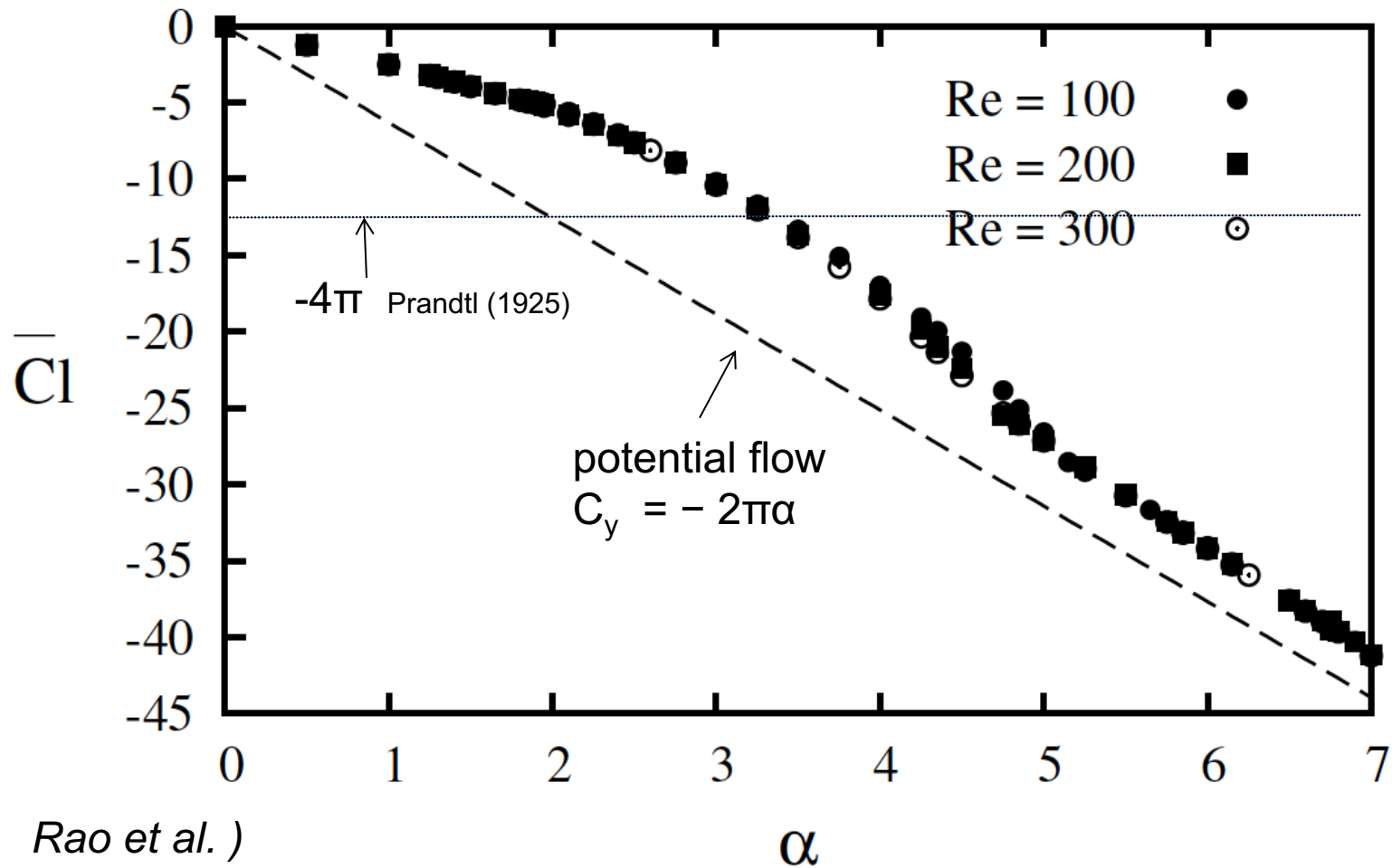
$Re=400$



*Kumar et al,  
JFE, 2011*

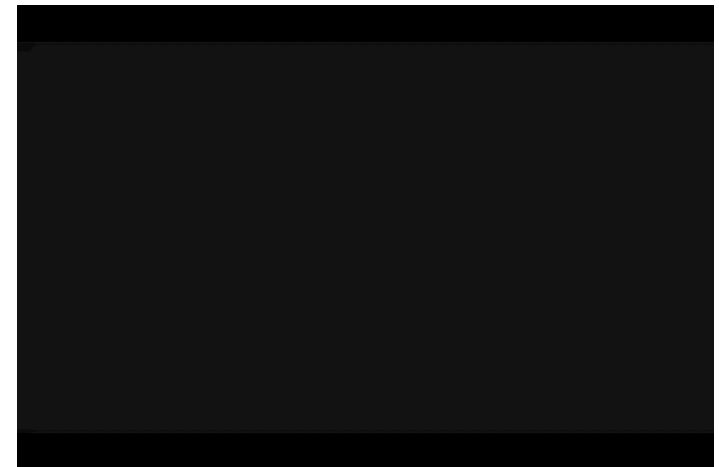
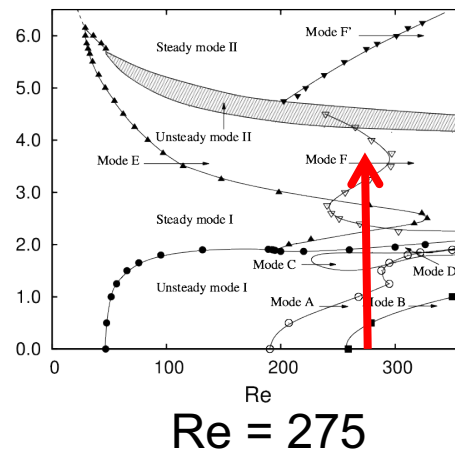
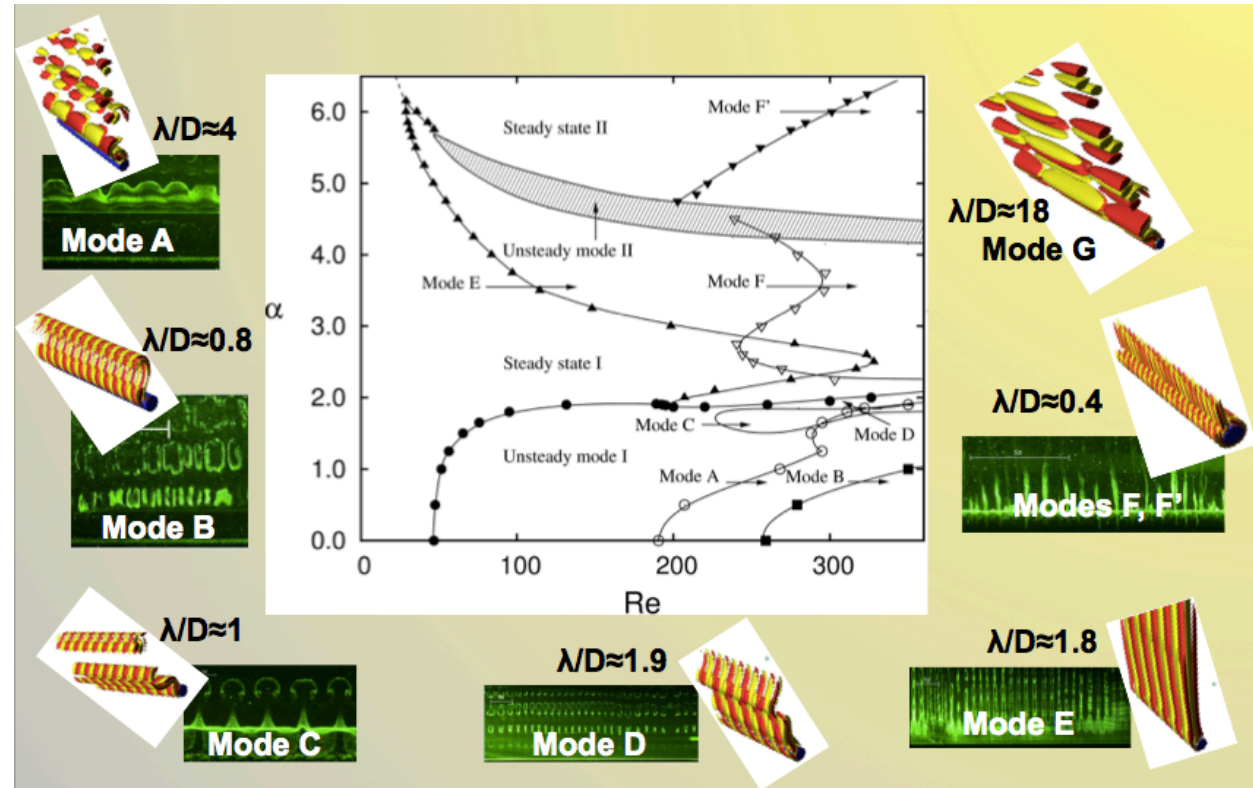


# Predicted lift force coefficient



# 3d Transition Parameter Map

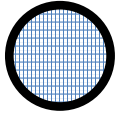
- Without rotation: modes A & B
- Increasing rotation adds many new 3d modes
- Flow becomes steady as  $\alpha > 2$ , then unsteady again for  $\alpha \approx 4-5$ , then steady again
- Movie shows modes as  $\alpha$  increases from 0 to 4, at  $Re = 275$



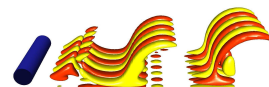
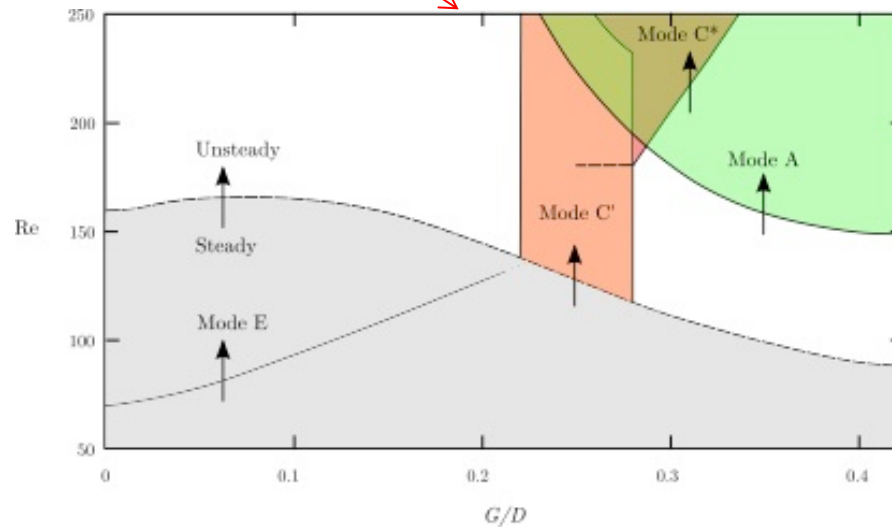
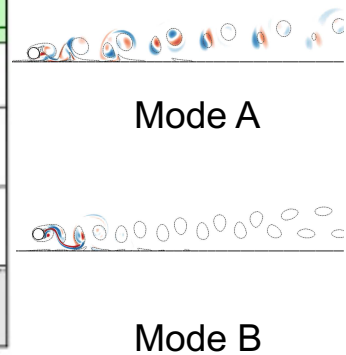
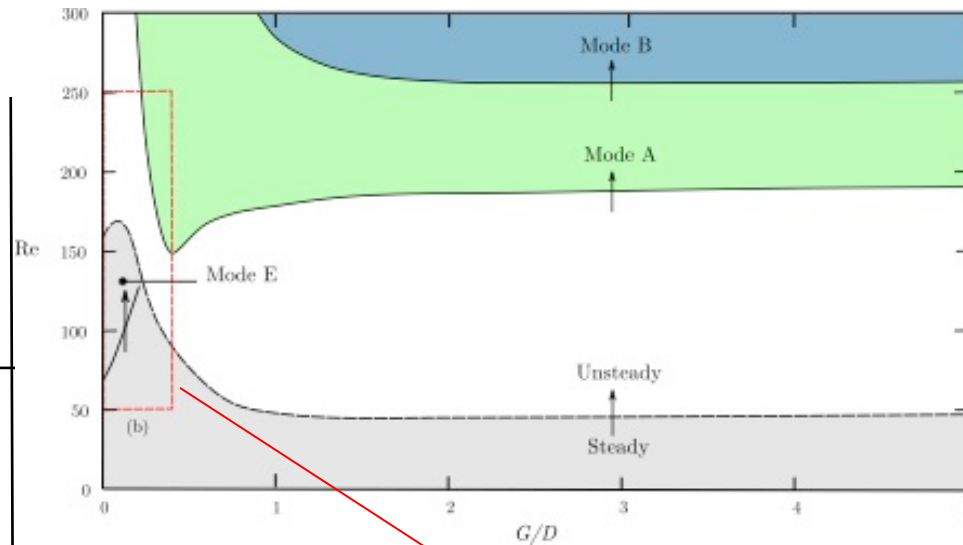
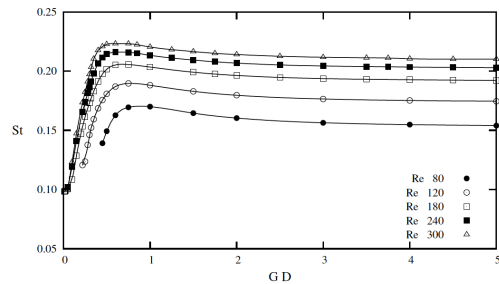
# Effect of a wall on wake structure and transitions

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# $\alpha = 0$ (sliding cylinder)



Mode	$\lambda/D$	Nature of $\mu$	Gap height	Base flow
A	$\approx 4$	Real and positive	$\geq 0.25D - \infty$	Unsteady
B	$\approx 0.8$	Real and positive	$\geq 0.85D - \infty$	Unsteady
C'	$\approx 1.25$	Real and negative	$\approx 0.22D - 0.28D$	Unsteady
C*	$\approx 1.5$	Real and negative	$\approx 0.28D - 0.35D$	Unsteady
E	$\approx 6$	Real and positive	$0 - 0.2D$	Steady

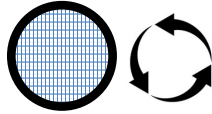


Mode C'

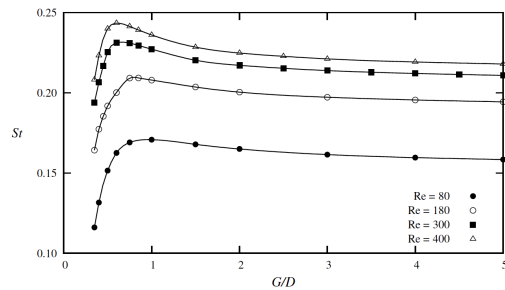


Mode C\*

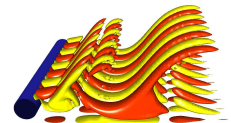
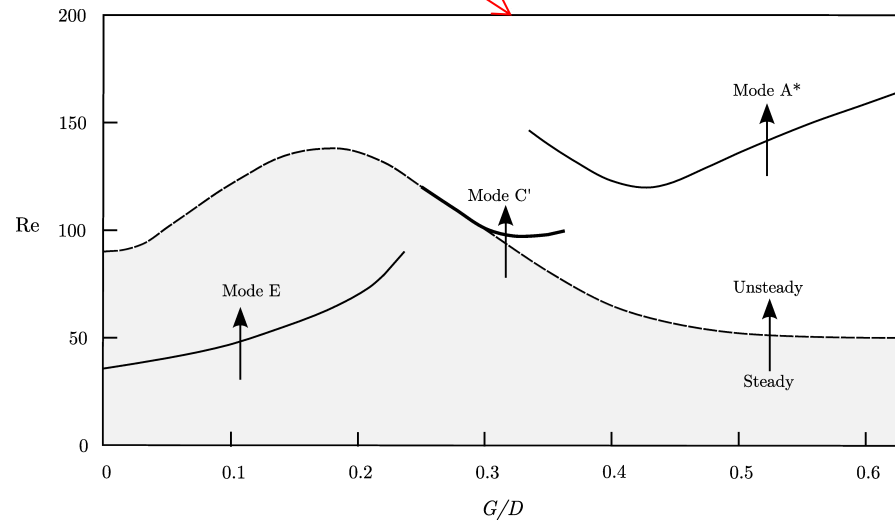
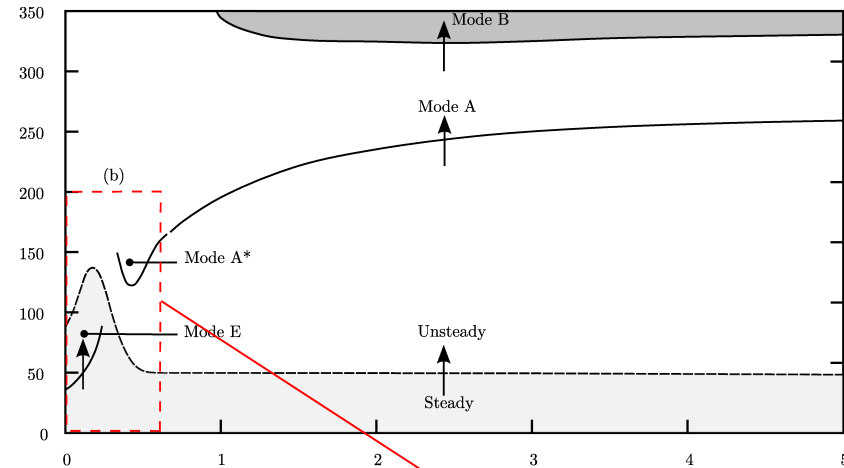
# $\alpha = +1$ (prograde rolling cylinder)



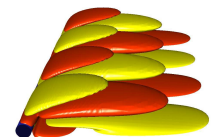
Mode	$\lambda/D$	Nature of $\mu$	Gap height	Base flow
A	$\approx 4$	Real and positive	$\gtrsim 0.75D - \infty$	Unsteady
A*	$\approx 4$	Real and positive	$\approx 0.35D - 0.6D$	Unsteady
B	$\approx 0.8$	Real and positive	$\gtrsim 1D - \infty$	Unsteady
C'	$\approx 2$	Real and negative	$\approx 0.25D - 0.35D$	Unsteady
E	[8 - 14]	Real and positive	0 - 0.22D	Steady



Re



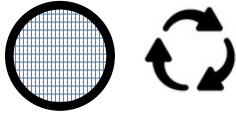
Mode C'



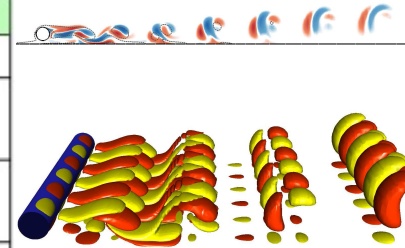
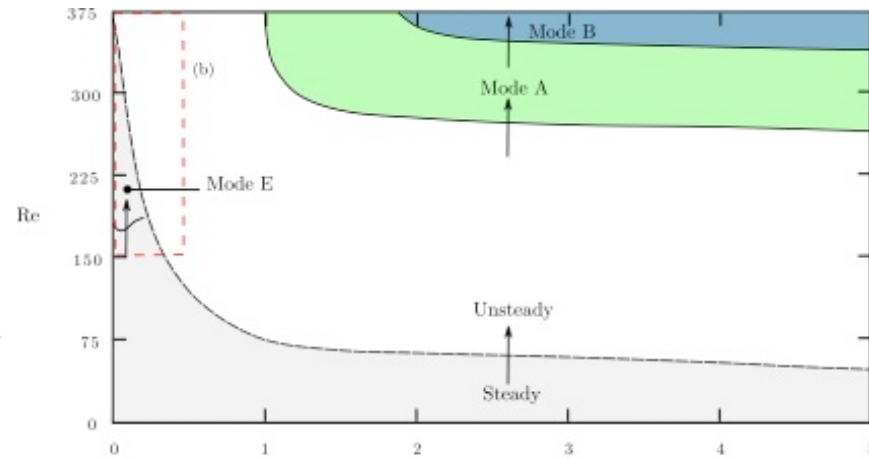
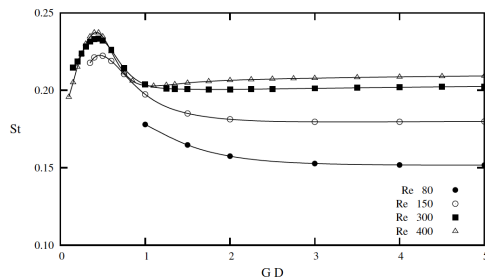
Mode E



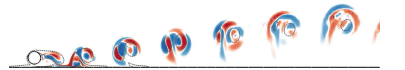
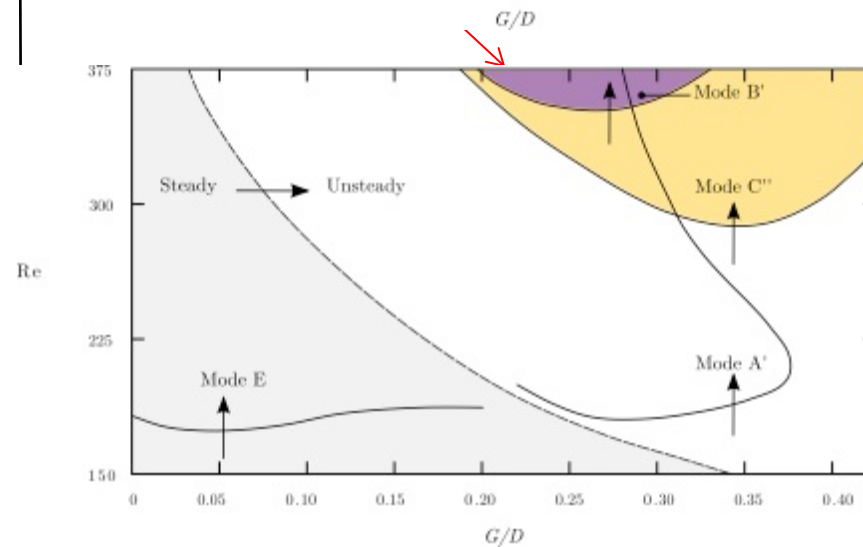
# $\alpha = -1$ (retrograde spin cylinder)



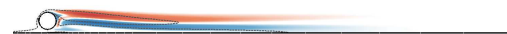
Mode	$\lambda/D$	Nature of $\mu$	Gap height	Base flow
A	$\approx 4$	Real and positive	$\gtrsim 1D - \infty$	Unsteady
A'	$\approx 2.4$	Real and positive	$\approx 0.22D - 0.37D$	Unsteady
B	$\approx 0.8$	Real and positive	$\gtrsim 1.75D - \infty$	Unsteady
B'	$\approx 0.77$	Real and positive	$\approx 0.22D - 0.32D$	Unsteady
C''	$\approx 1.5$	Real and negative	$\gtrsim 0.22D - 0.45D$	Unsteady
E	$\approx 2.4$	Real and positive	$0 - 0.2D$	Steady



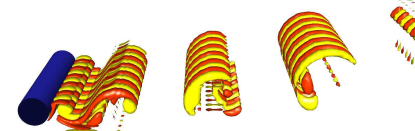
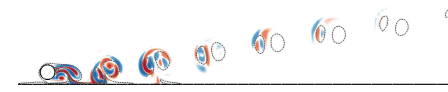
Mode A'



Mode C''

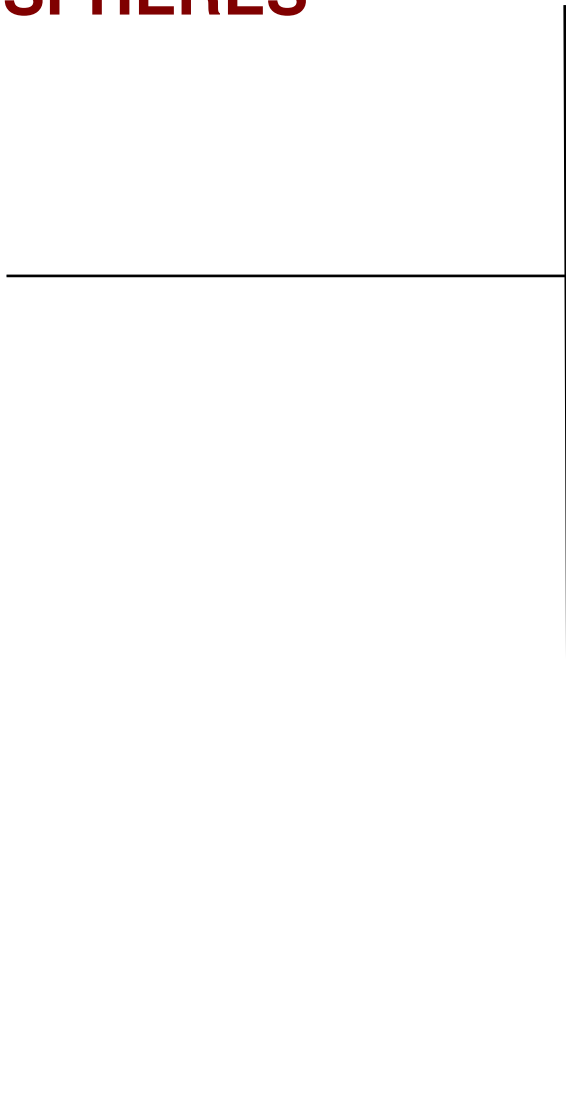


Mode E



Mode B'

# SPHERES



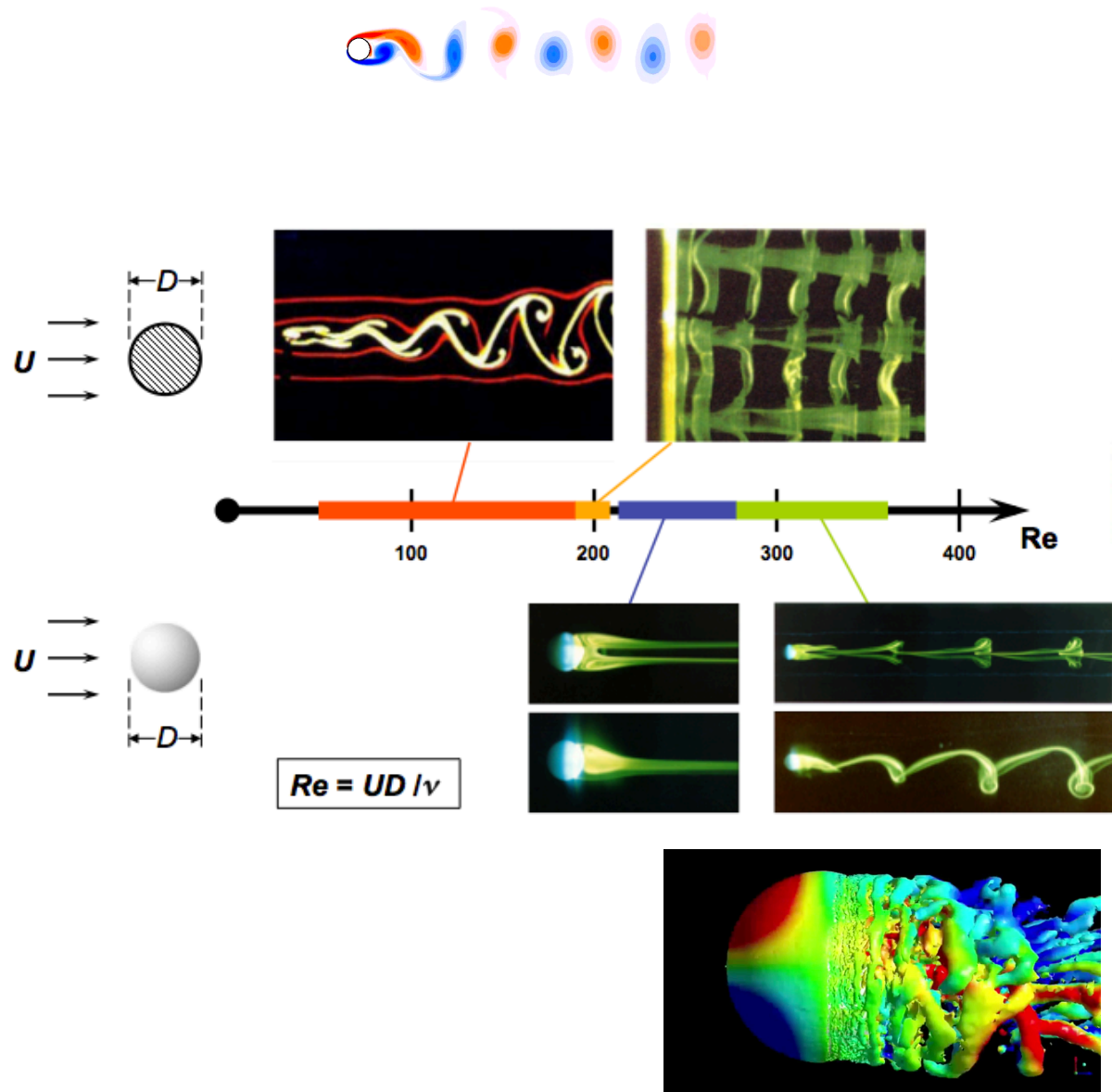
# Isolated cylinder and sphere have important wake transitions at moderate Re

## Cylinder

- Unsteady 2s at  $Re = 47$
- 3d mode A at  $Re = 180$
- 3d mode B at  $Re = 250$

## Sphere

- Steady asymmetrical at  $Re = 212$
- Unsteady symmetrical at  $Re = 272$  (Hopf)

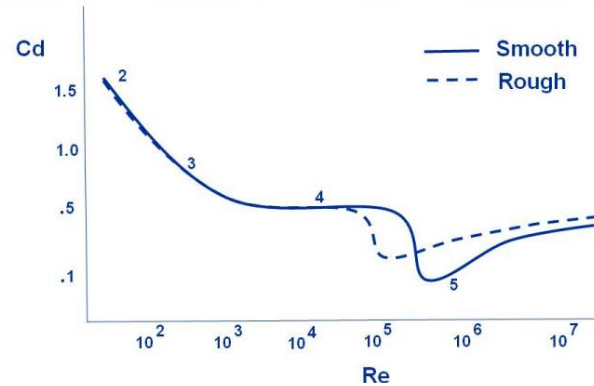


# Sphere wake transitions

- Strouhal number and drag coefficient variation
- Initial transitions in low-moderate Re range

National Aeronautics and Space Administration

## Drag of a Sphere



Drag coefficient and reciprocal of Strouhal number.  
NASA

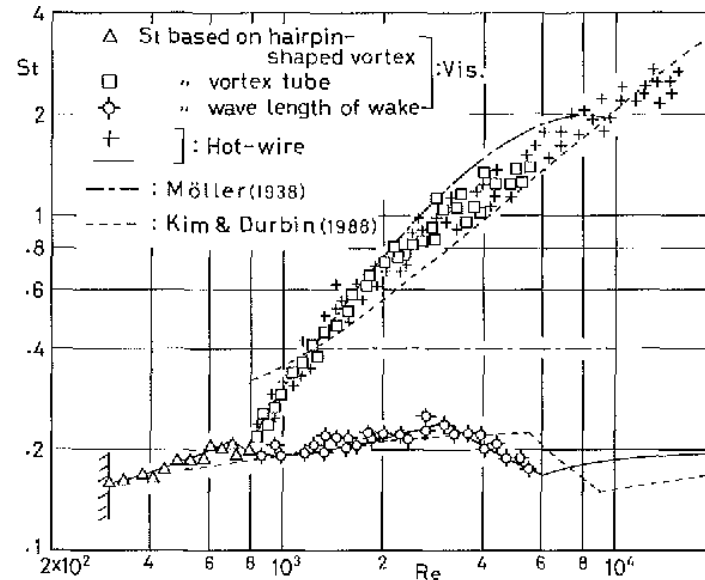


Fig. 4 Distribution of low-mode and high-mode Strouhal number versus:

Strouhal number vs Reynolds number

Sakamoto & Haniu, *J. Fluids Eng* 112(4), 386-392 (Dec 01, 1990)

# Non-Rotating spheres

- Johnson & Patel, JFM, 1999.

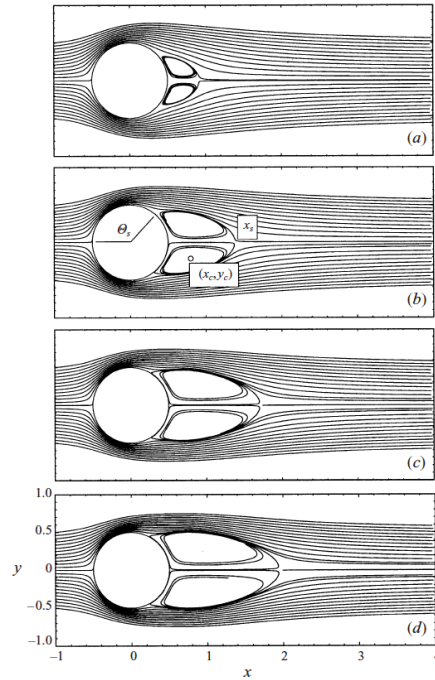


FIGURE 3. Computed axisymmetric streamlines past the sphere: (a)  $Re = 50$ ; (b)  $Re = 100$ ; (c)  $Re = 150$ ; (d)  $Re = 200$ .

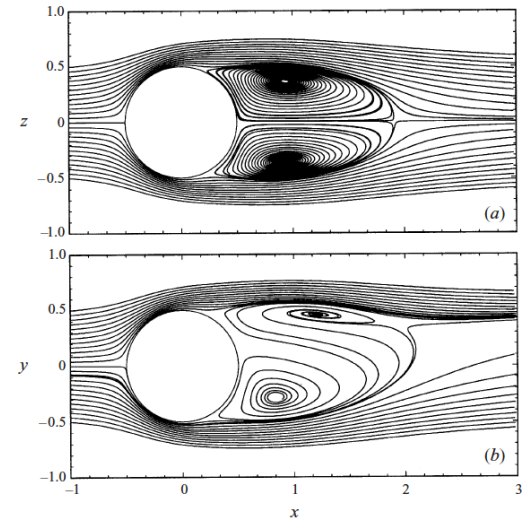


FIGURE 11. Streamlines of projected velocity vectors at  $Re = 250$  in (a) the  $(x, z)$ -plane; (b) the  $(x, y)$ -plane.

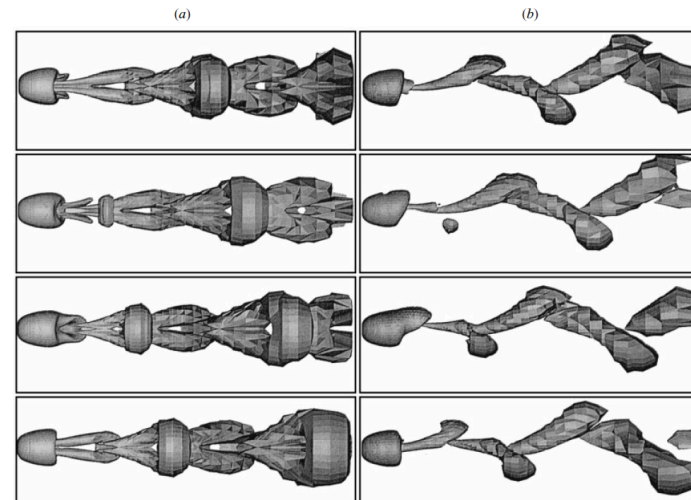


FIGURE 30. Vortical structure at  $Re = 300$  at every quarter period: (a)  $x, z$  view; (b)  $x, y$  view.

# Non-Rotating spheres

- Wake at  $Re=300$

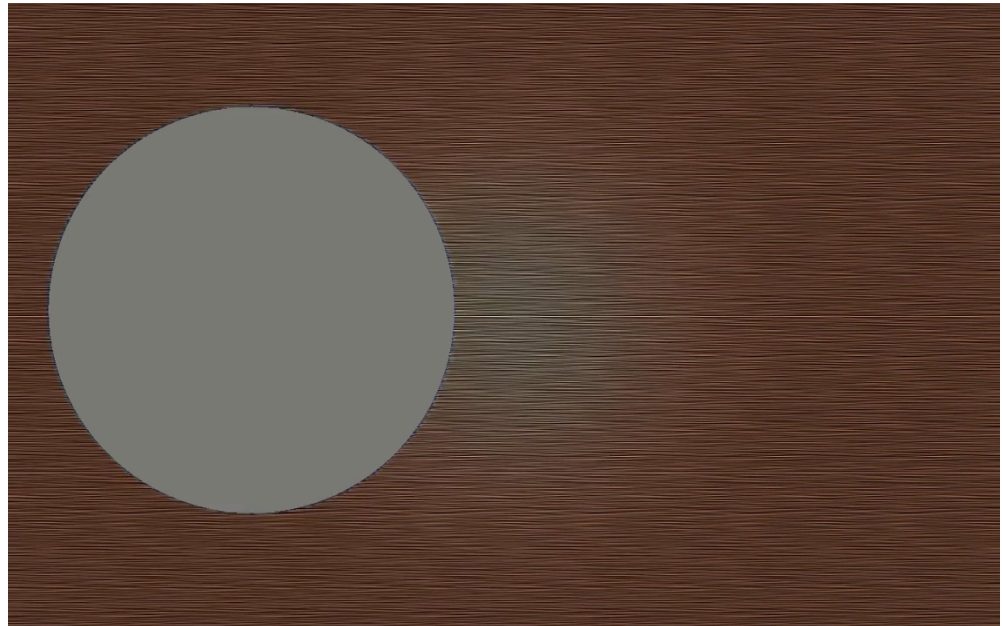
ST. ANTHONY  
FALLS LABORATORY  

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UNIVERSITY OF MINNESOTA

# Non-Rotating spheres

- Wake at  $Re=10,000$



# Rotating spheres

- Giacaobello et al., JFM, 2009.

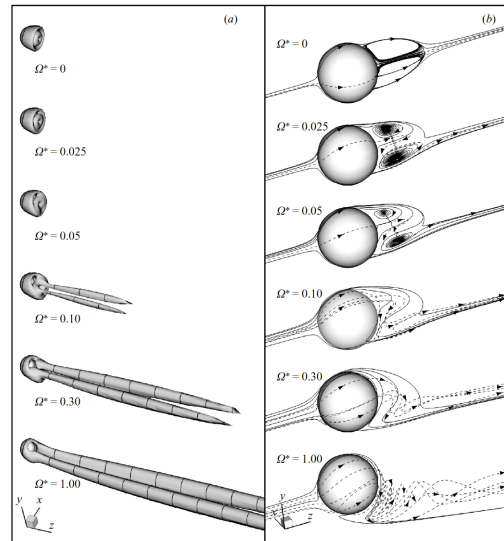
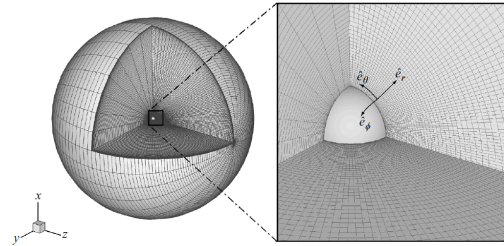


FIGURE 6. (a) Vortical structures and (b) three-dimensional streamline patterns for  $Re = 100$ .

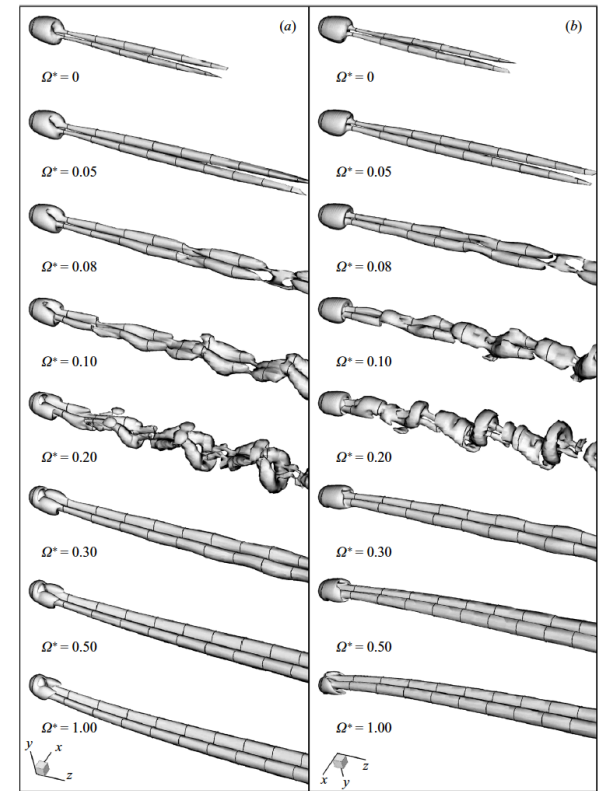


FIGURE 7. Instantaneous vortical structures for  $Re = 250$ : (a) and (b) look down on the wake from the retreating and advancing side of the sphere, respectively.

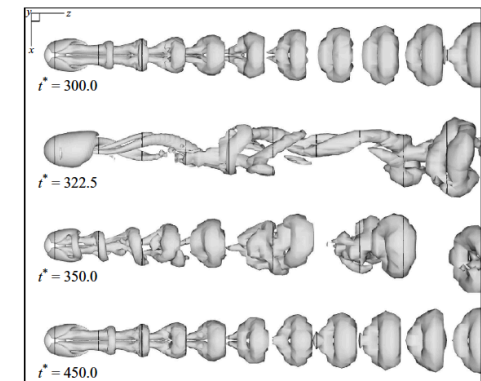


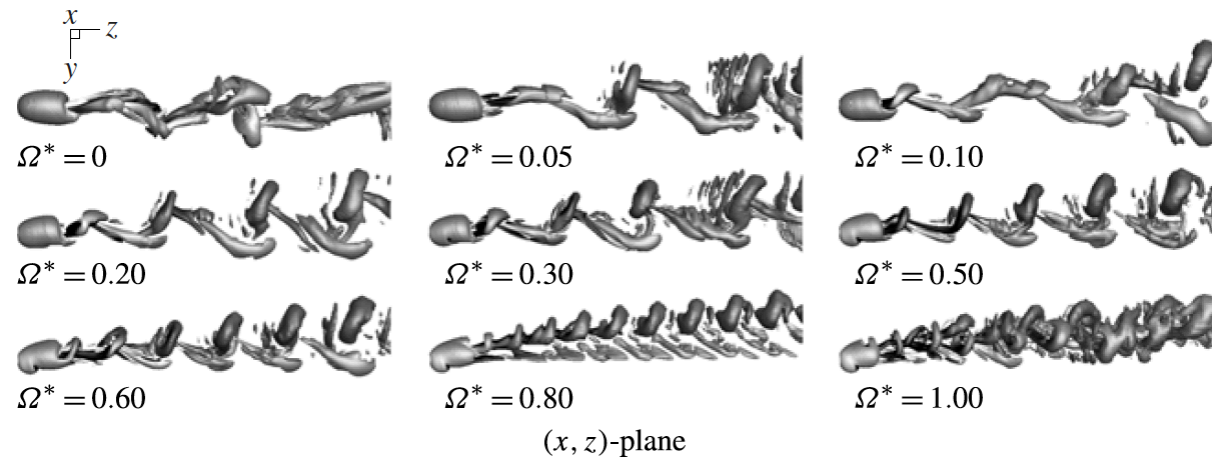
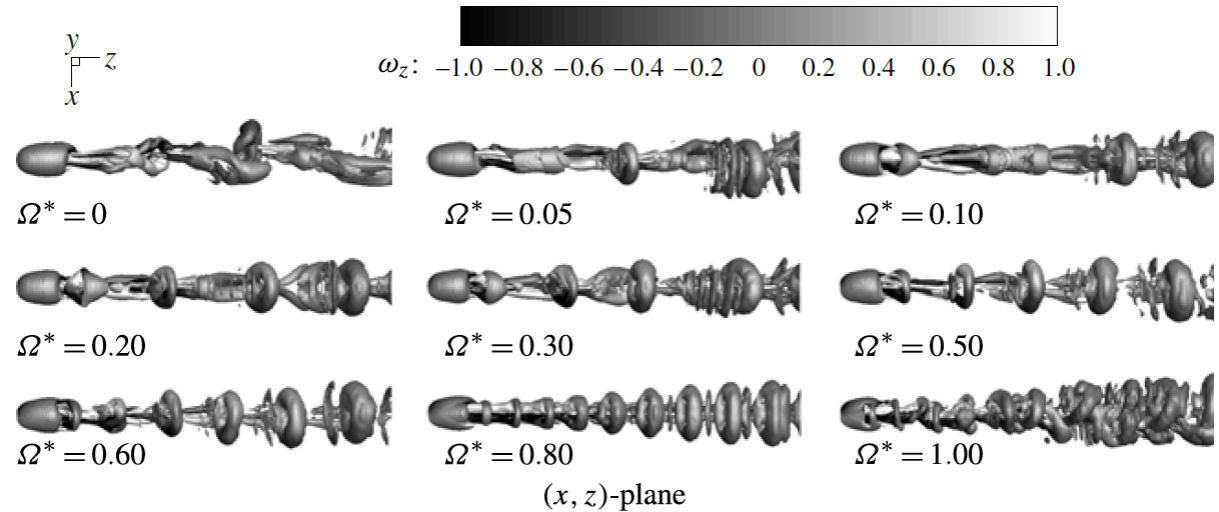
FIGURE 14. Effect of symmetry breaking perturbation for  $Re = 300$  and  $\Omega^* = 1.00$ . The time sequence begins with the statistically steady flow for transversely rotating sphere at  $t^* = 300.0$ . This is followed by streamwise rotation for  $300.0 < t^* \leq 322.5$  and transverse rotation for  $t^* > 322.5$ . Vortical structures are identified by plotting isosurfaces of  $\lambda_2 = -8 \times 10^{-4}$ .



# Rotating spheres

- $Re = 500$
- Variation of wake with increasing spin rate
- Streamwise vorticity plotted ( $\lambda_2$ )

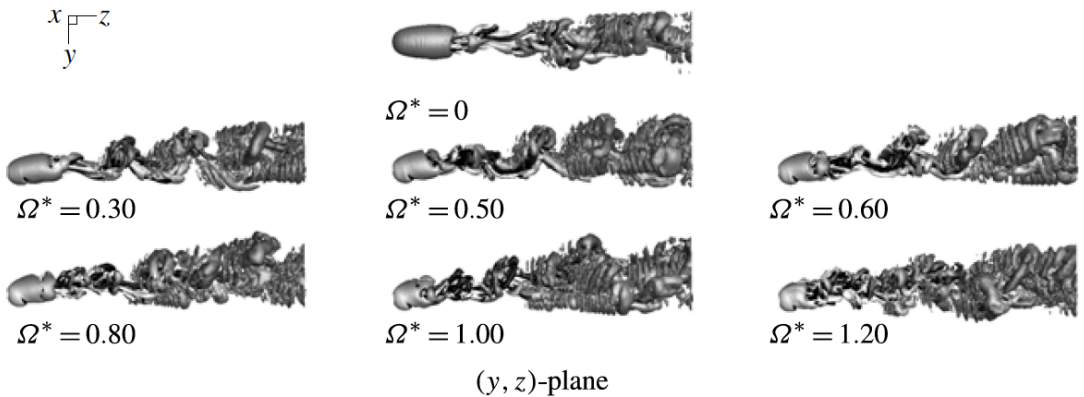
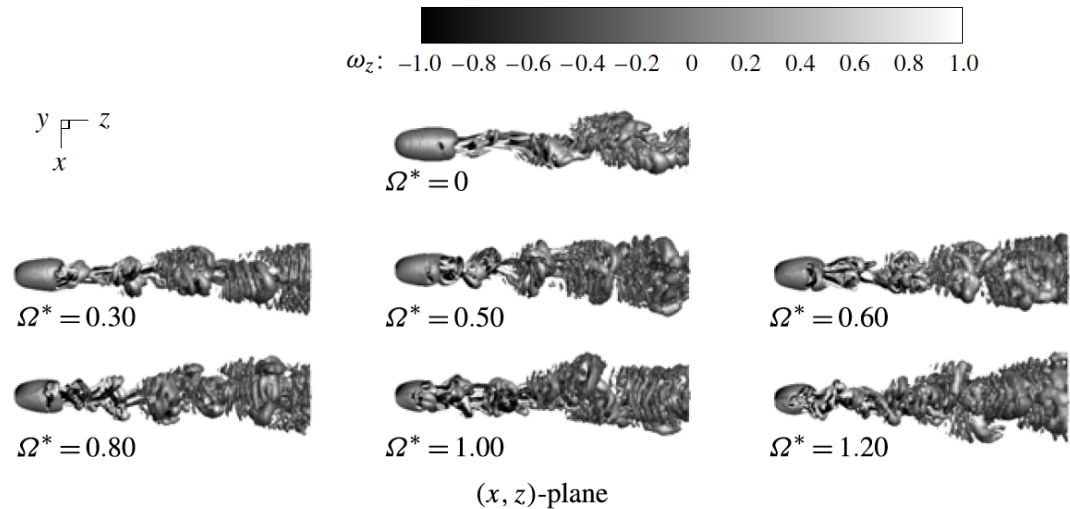
Poon et al., 2015.



# Rotating spheres

- $Re = 1000$
- Variation of wake with increasing spin rate
- Streamwise vorticity plotted ( $\lambda_2$ )

Poon et al., 2015.



# Rotating spheres

- State of wake can change if computed for longer times for  $\alpha = 1$  and 1.5.
- Rajamuni et al., JFM, 2018a
- $Re = 300$
- cf Giacobello et al., 2009.

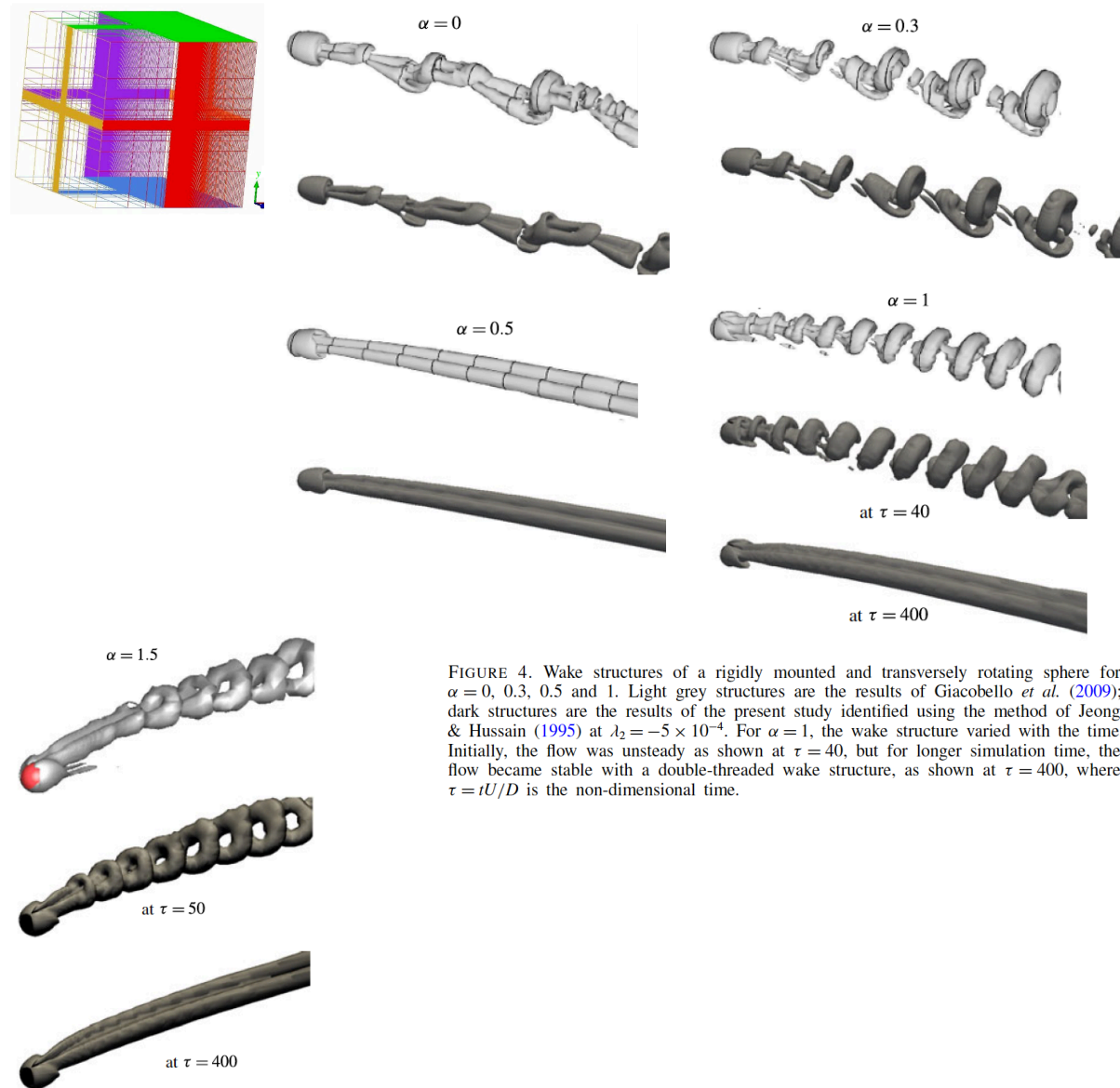


FIGURE 4. Wake structures of a rigidly mounted and transversely rotating sphere for  $\alpha = 0, 0.3, 0.5$  and 1. Light grey structures are the results of Giacobello *et al.* (2009); dark structures are the results of the present study identified using the method of Jeong & Hussain (1995) at  $\lambda_2 = -5 \times 10^{-4}$ . For  $\alpha = 1$ , the wake structure varied with the time. Initially, the flow was unsteady as shown at  $\tau = 40$ , but for longer simulation time, the flow became stable with a double-threaded wake structure, as shown at  $\tau = 400$ , where  $\tau = tU/D$  is the non-dimensional time.

FIGURE 5. (Colour online) Comparison of wake structures of a rigidly mounted and transversely rotating sphere for  $\alpha = 1.5$  with Dobson *et al.* (2014) (the dark structure). The light structures are the results of the present study.

# Applications of rotating cylinders/spheres

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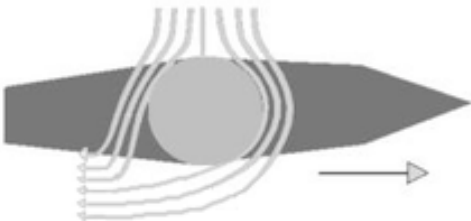
- Sails
- Wings
- Sports
- Wind turbines

# Flettner Rotors – rotating round sails provide propulsion

- 



Flettner Rotor Ship  
1920



[http://www.youtube.com/watch?v=\\_\\_8-QSXgupA](http://www.youtube.com/watch?v=__8-QSXgupA)



<http://www.youtube.com/watch?v=2pQga7jxAyc>



# Flettner Rotors – rotating round wings provide lift



Anton Flettner's  
Rotor Aircraft (1930)



<http://www.youtube.com/watch?v=hlmvHfIASzo>

## Rotating Cylinder Helicopter Rotor

Nathan Phillips  
Undergraduate Thesis

Ryerson University  
Department of Aerospace Engineering  
April 2006

# Sports – Magnus effect is important

- Many sports take advantage of Magnus Effect: tennis, golf, table tennis, football, baseball, cricket, etc
- Magnus effect on ball
- “Impossible” Goal



[http://www.youtube.com/watch?feature=player\\_detailpage&v=3ECor\\_tJNQ](http://www.youtube.com/watch?feature=player_detailpage&v=3ECor_tJNQ)

Roberto Carlos



# Wind Turbines - Transverse Axis

- Cylinder (chimney) suffers FSI oscillations

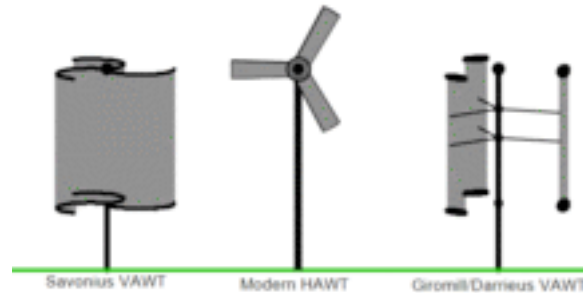
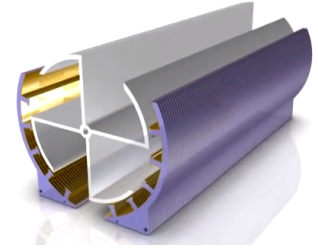


- Add rotation, and larger oscillations over wide velocity range
- Rotation can cause high bending moments on blades & shaft of turbines



<http://www.youtube.com/watch?v=BybfnSoluY#t=17>

► New concept open-type wind generator



[http://en.wikipedia.org/wiki/Wind\\_turbine](http://en.wikipedia.org/wiki/Wind_turbine)





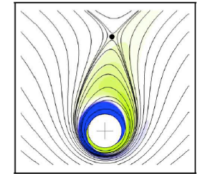
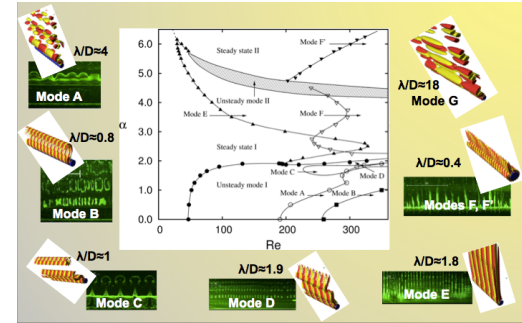
# Conclusions

Rotation of a **cylinder** in a flow:

1. Introduces many new wake transitions

2. Can suppress vortex shedding

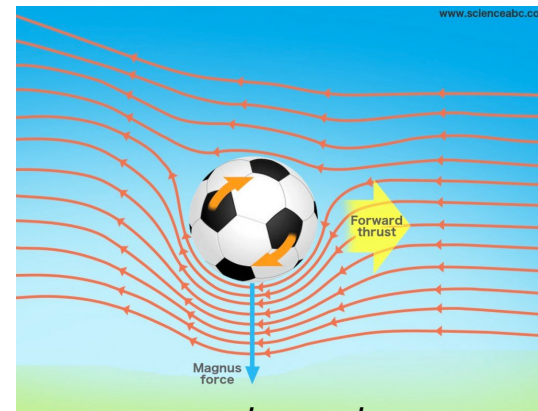
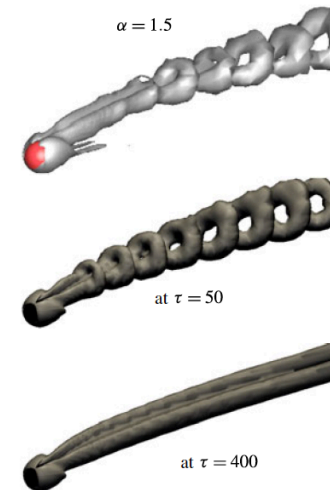
3. Increase dramatically lift and reduce drag



# Conclusions

Rotation of a **sphere** in a flow:

1. Alters the wake structure
2. Can suppress vortex shedding
3. Produces a net lift force





# Acknowledgements



National Computational Infrastructure



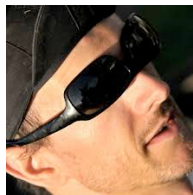
Dr Anirudh Rao



Dr Thomas Leweke  
CNRS/Aix-Marseille



Dr Alex Radi



Dr Justin Leontini  
Swinburne University



Prof Mark Thompson



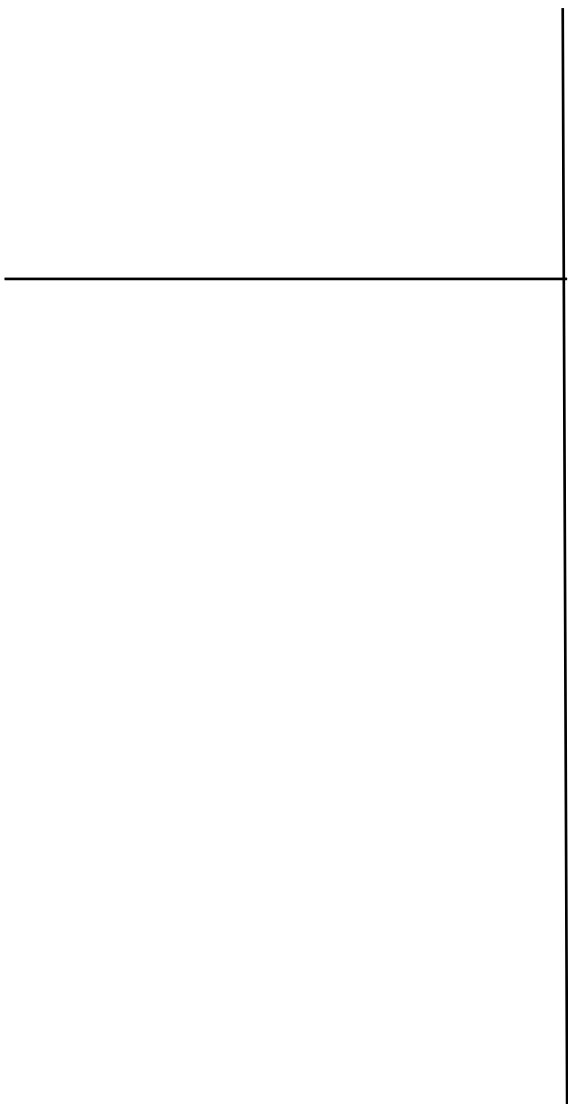
Dr David Lo Jacono  
IMFT, Toulouse

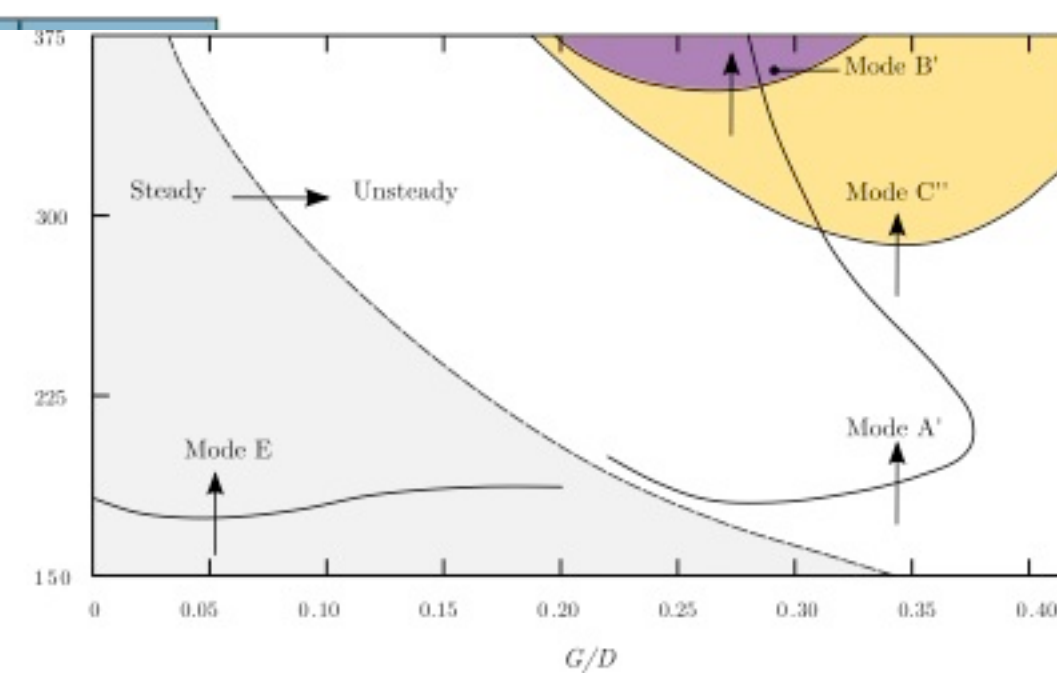
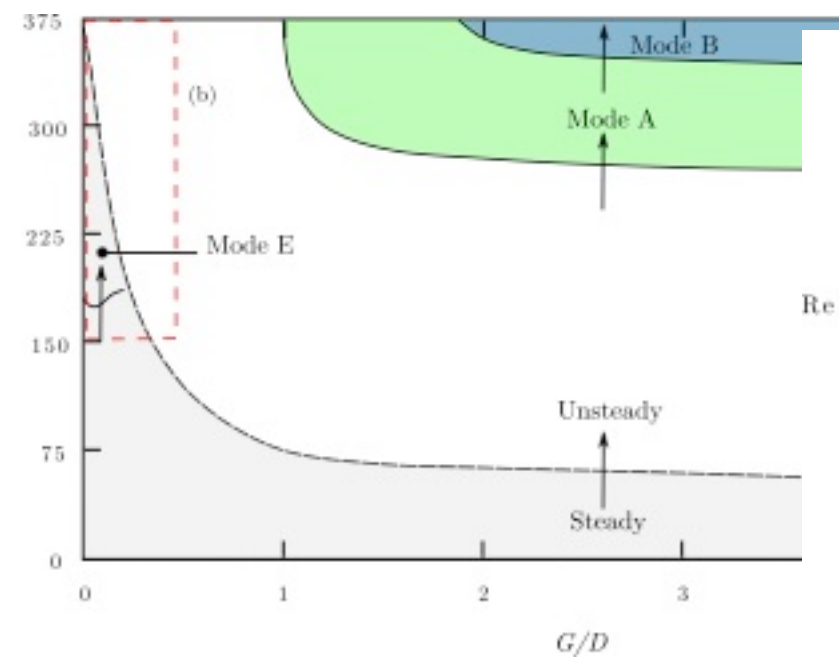
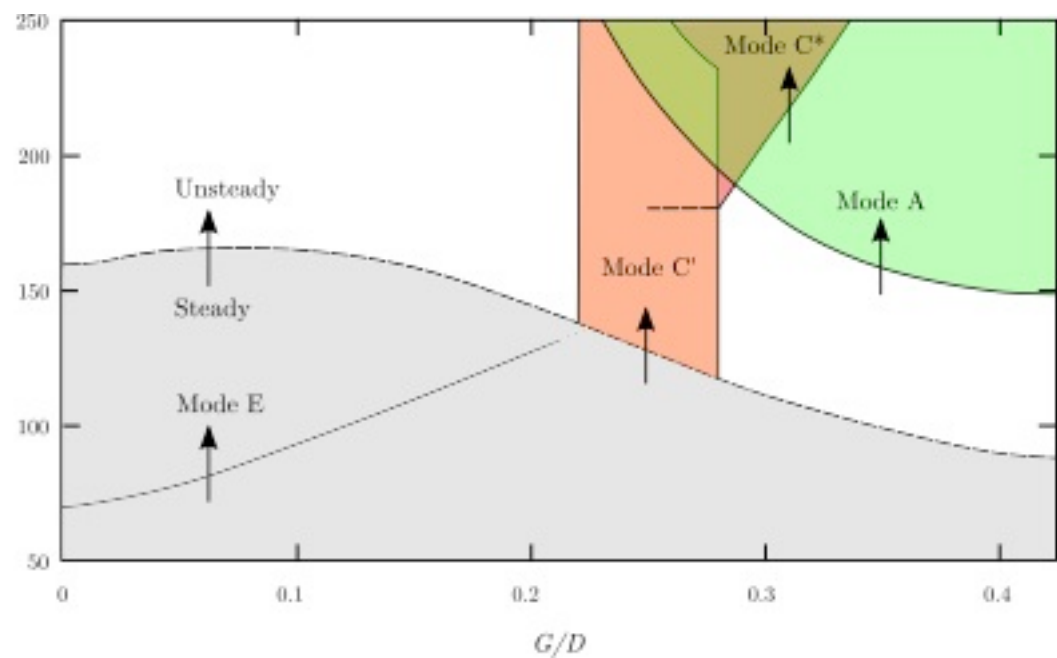
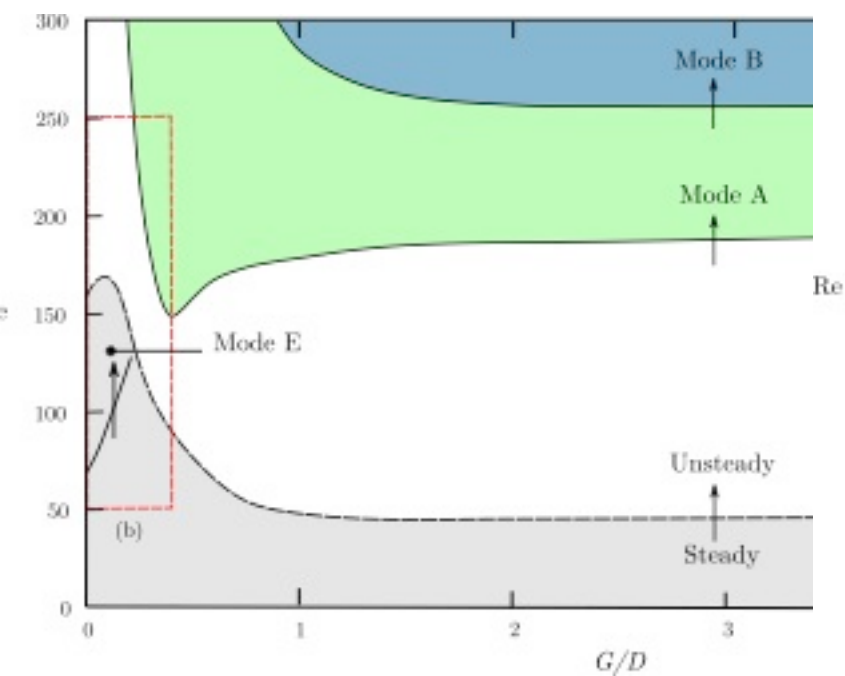


Prof John Sheridan

Thank you



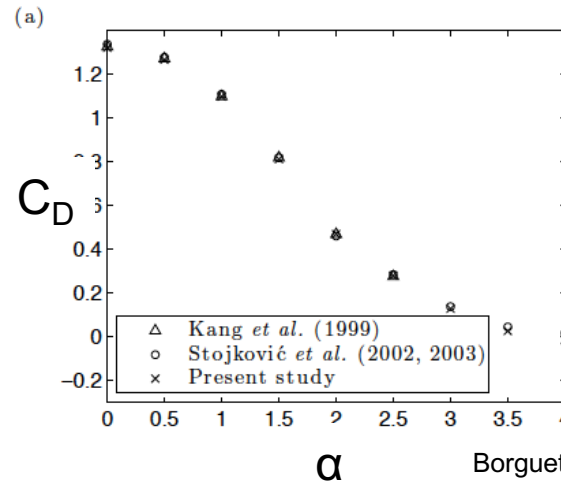




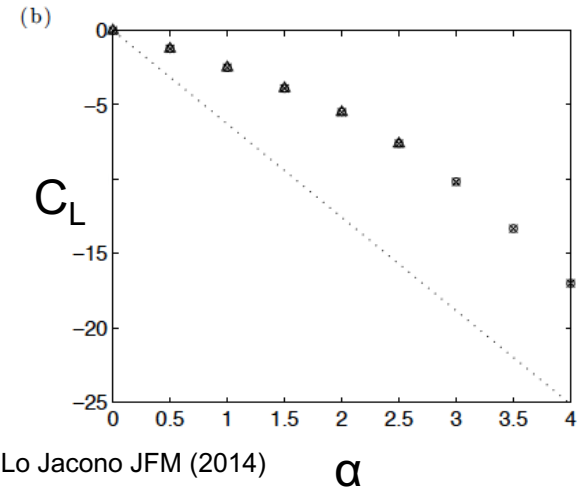
# Fluid-Structure Interaction

- Faster rotation =>
- Low (even negative!) drag
- High lift
- Elastically mount =>
- Larger oscillations
- Wide range of velocities

Fixed



Borguet & Lo Jacono JFM (2014)



Elastically mounted

