The Aerodynamics of Wind Turbines

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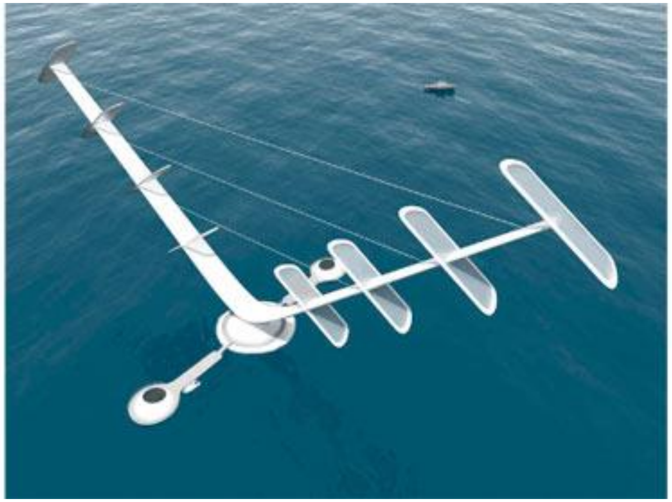
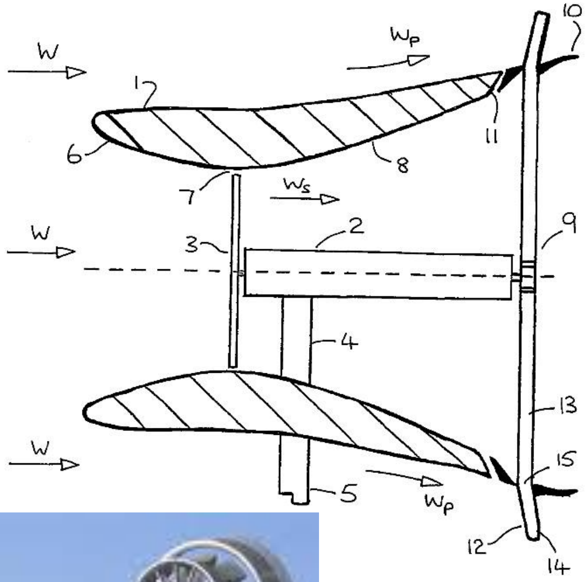
Wind Energy

Various wind turbines :

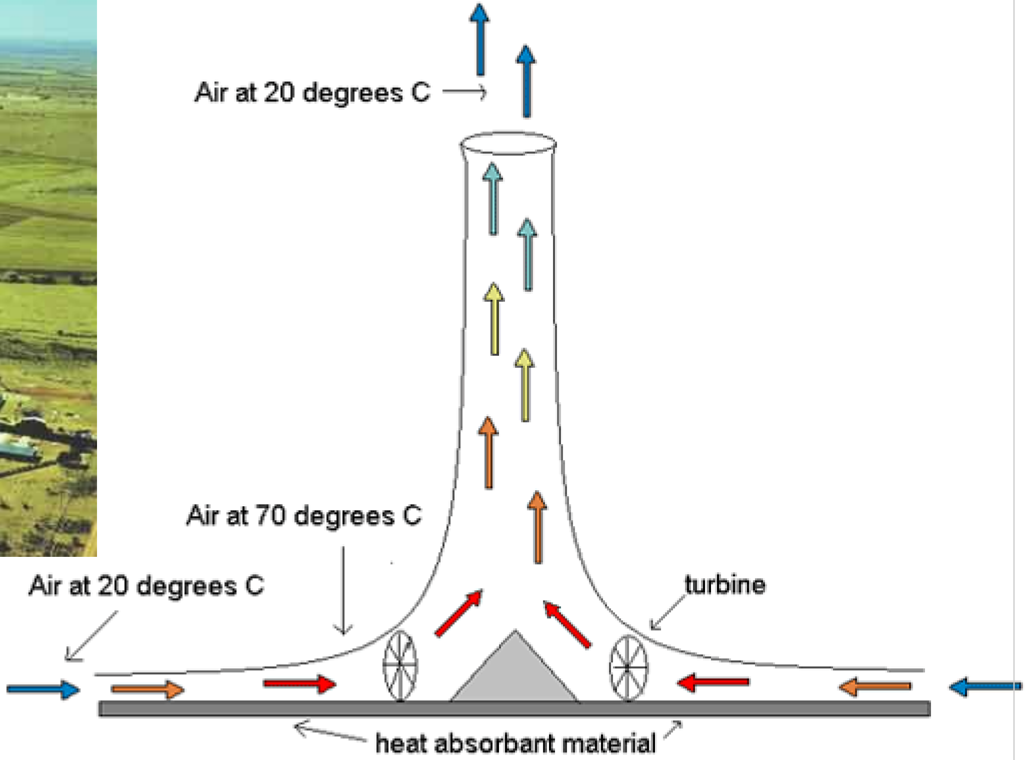
Alternative concepts



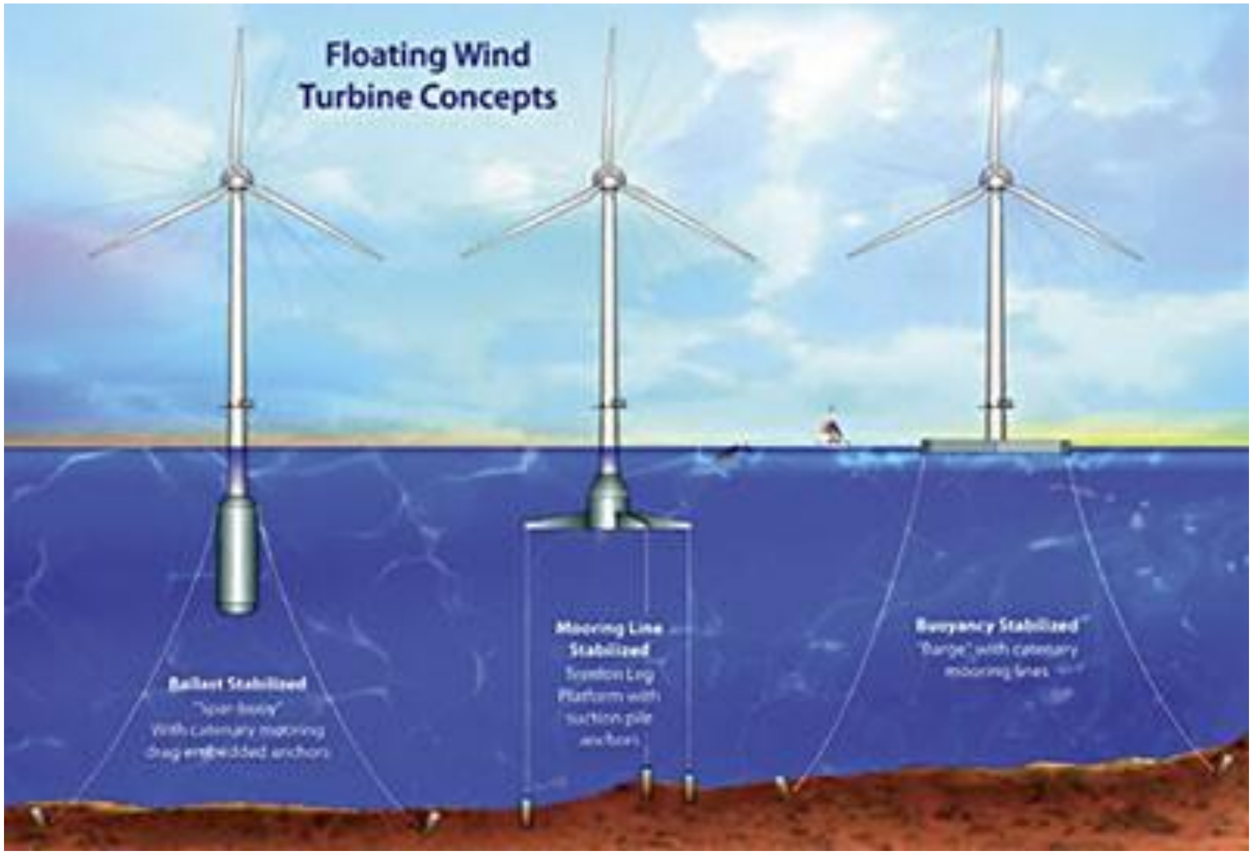
Diffuser Augmented Wind Turbines



Solar Chimney



Floating Wind Turbines



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The Danish Concept:

3-bladed upwind machine with gearbox and asynchroneous generator

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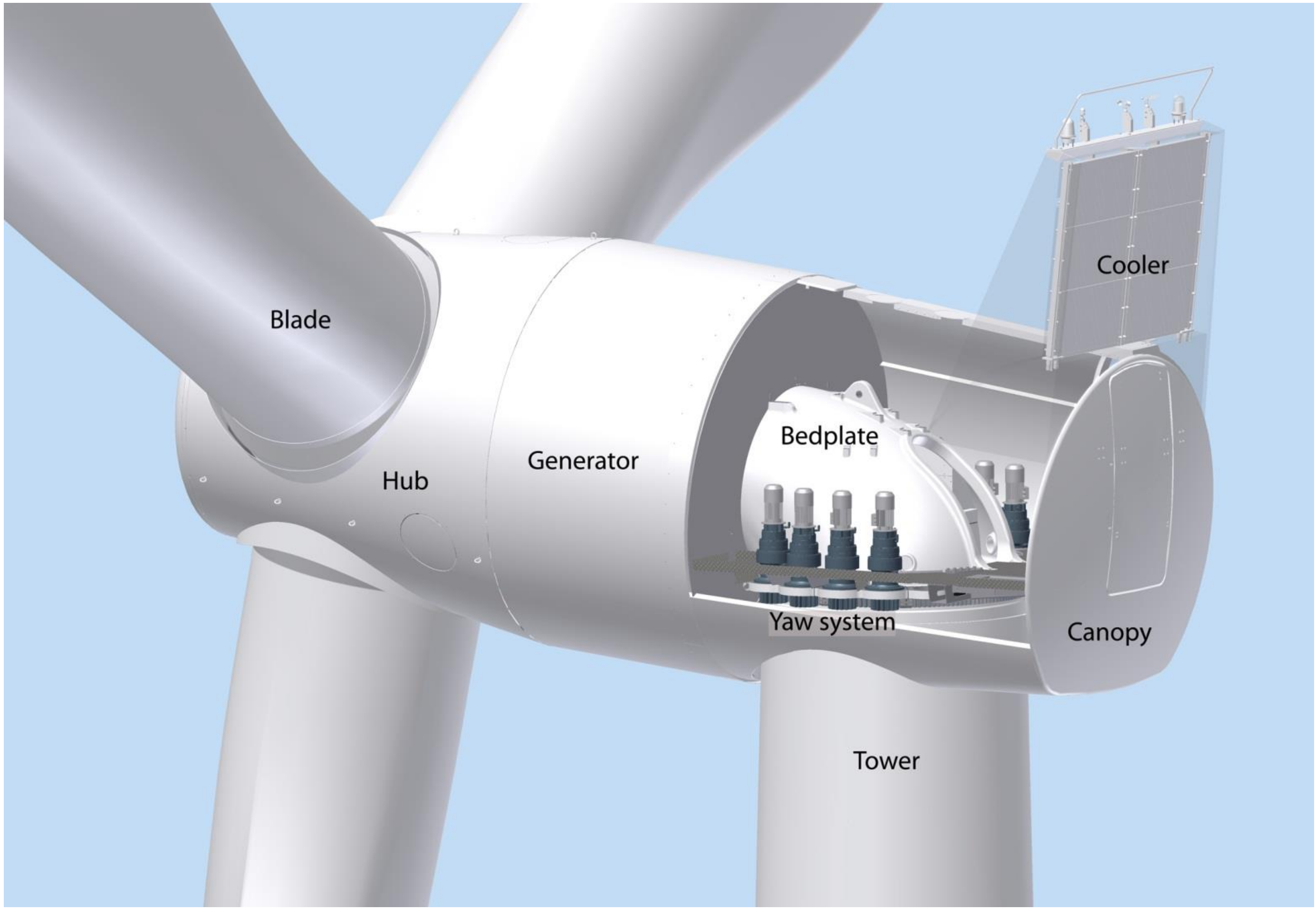
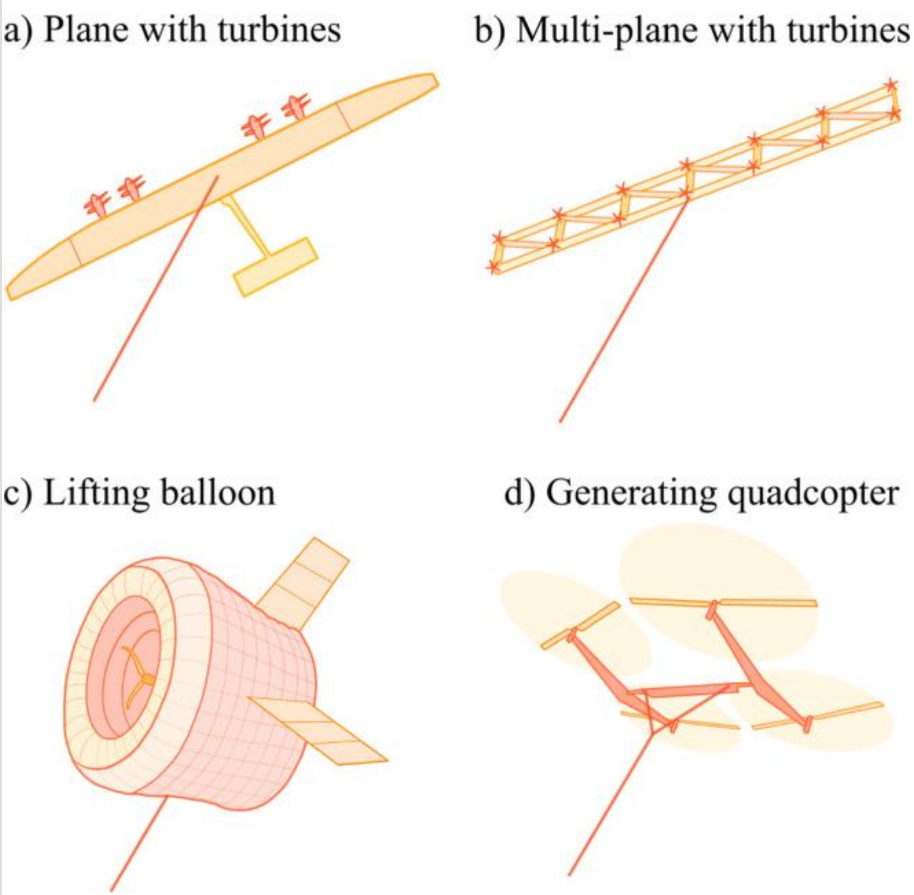
Example of gearbox-free turbine
Enercon 126: P=6MW; D=126m

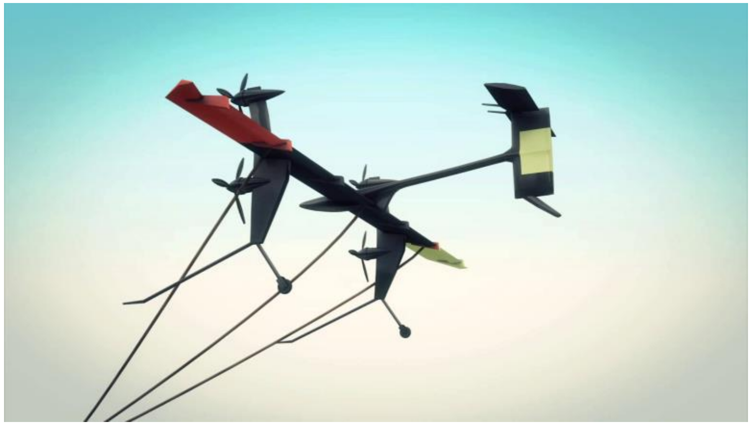
New Siemens Wind Turbine:SWT-7.0-154



Mitsubishi 7MW (81.6 m) wind turbine blade

Airborne wind turbines

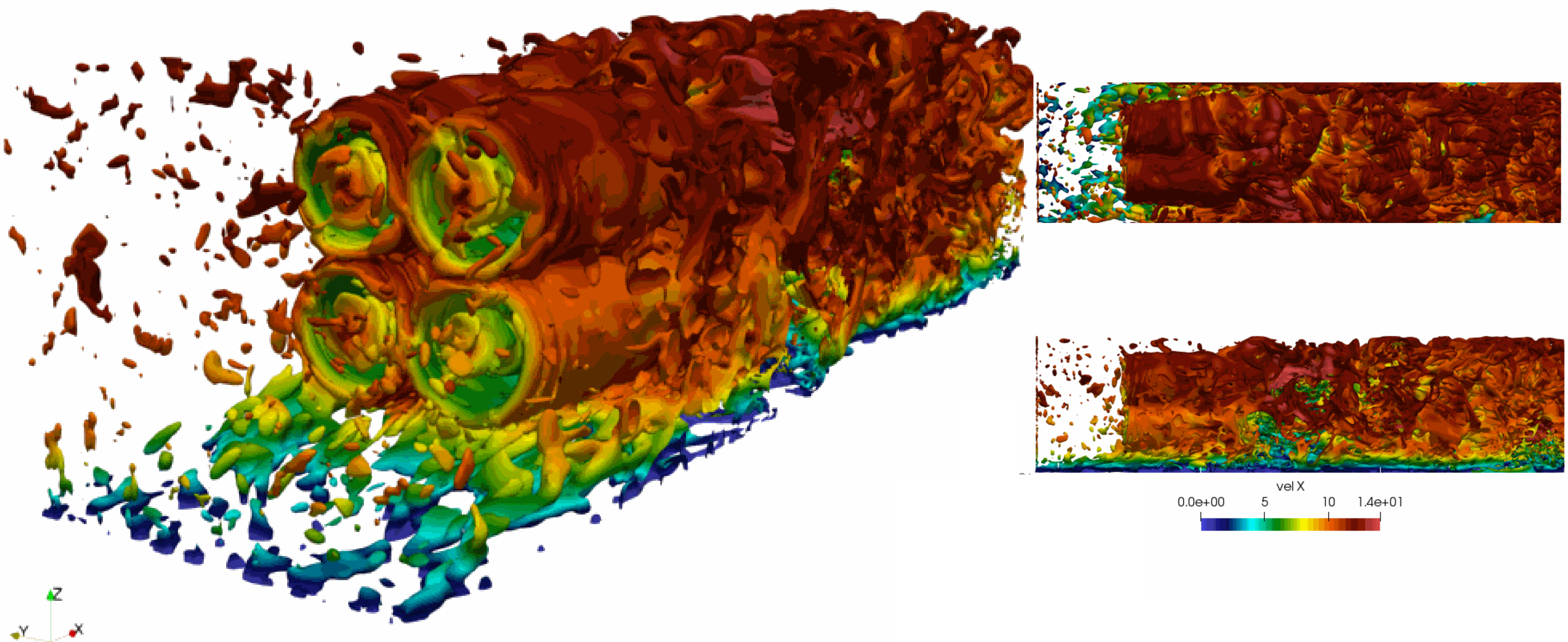




New 4-rotor Vestas Wind Turbine

Multi-rotor / lifting line particle wake

FULL AEROELASTIC SIMULATION



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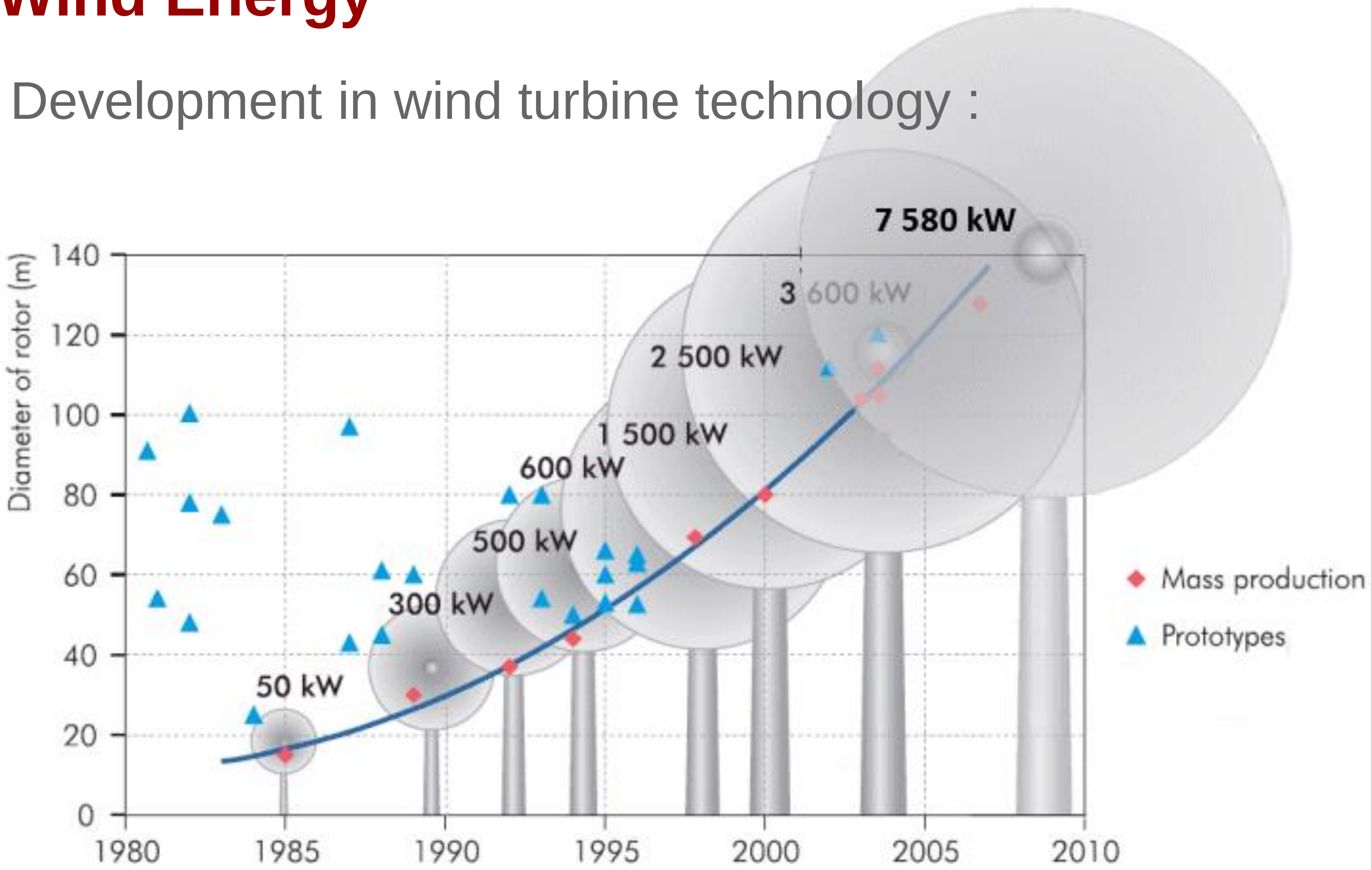
Development in wind turbine technology :



Source: International Energy Agency (IEA)

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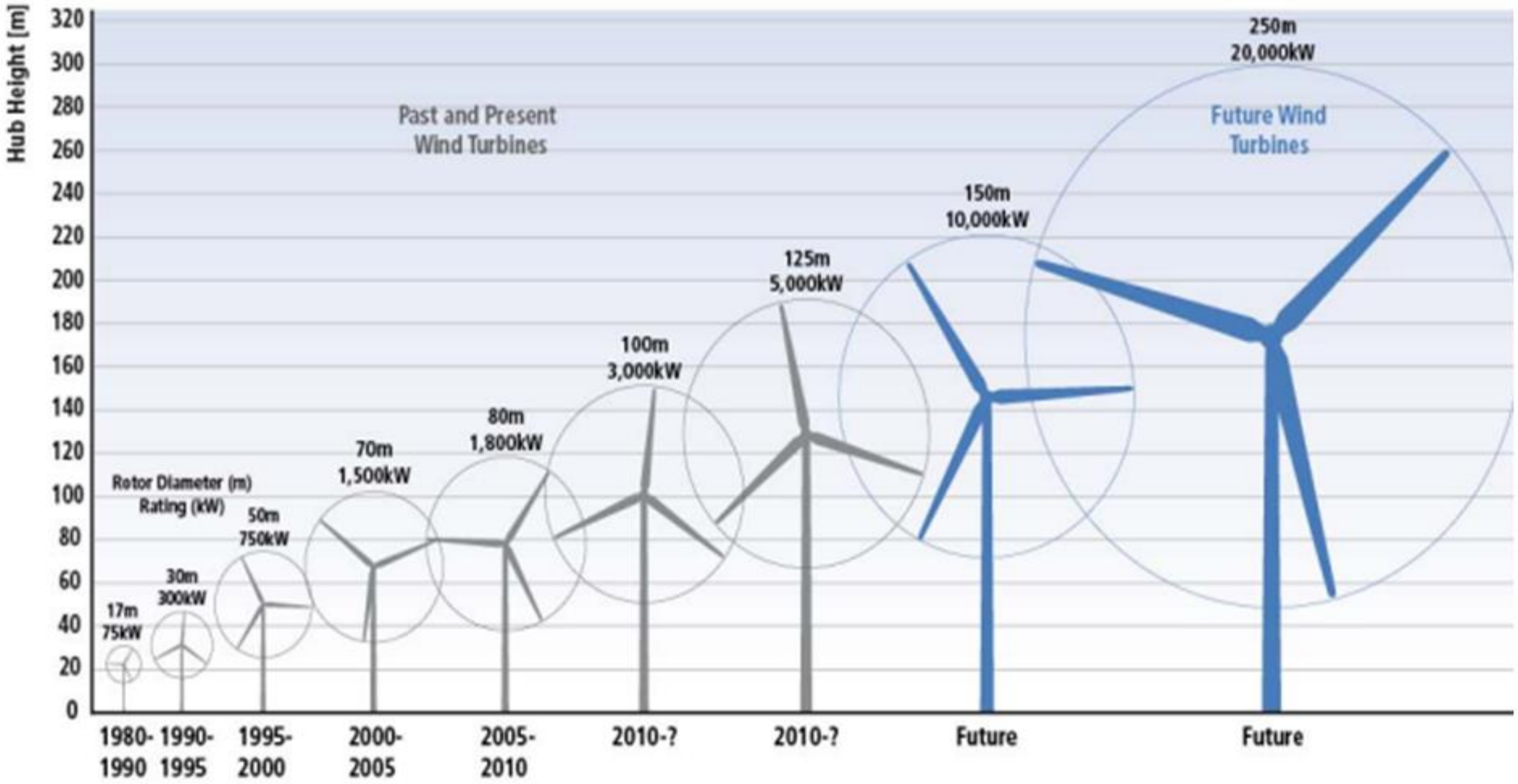
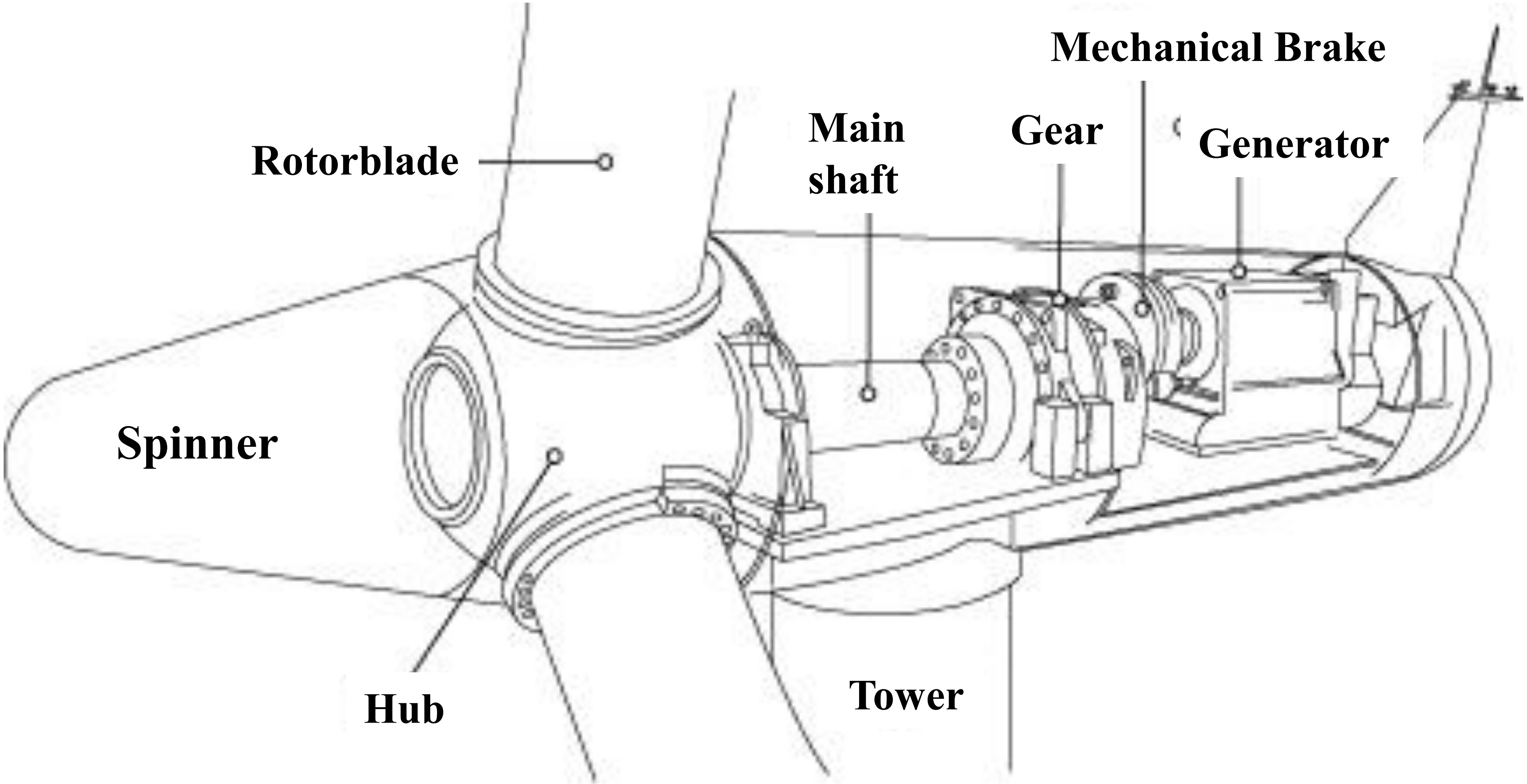
Technical Advancements: For instance growth in size of typical commercial wind turbines.



(from the IPCC April 2012 report on mitigating climate change)

Wind turbine

Nacelle :



Important Research Areas in Wind Power

Nature
- Wind Resources
- Wind/wave loading
- Environmental aspects
- Climatic aspects

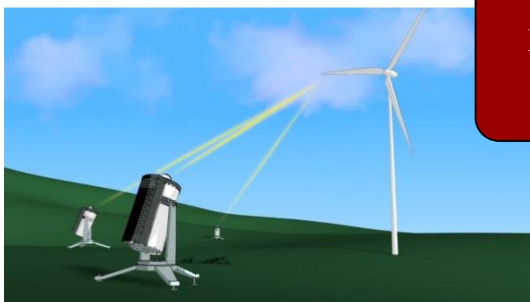
Wind Turbine Technology
- Aerodynamics/aeroelasticity
- Machinery elements
- Generators
- Foundation

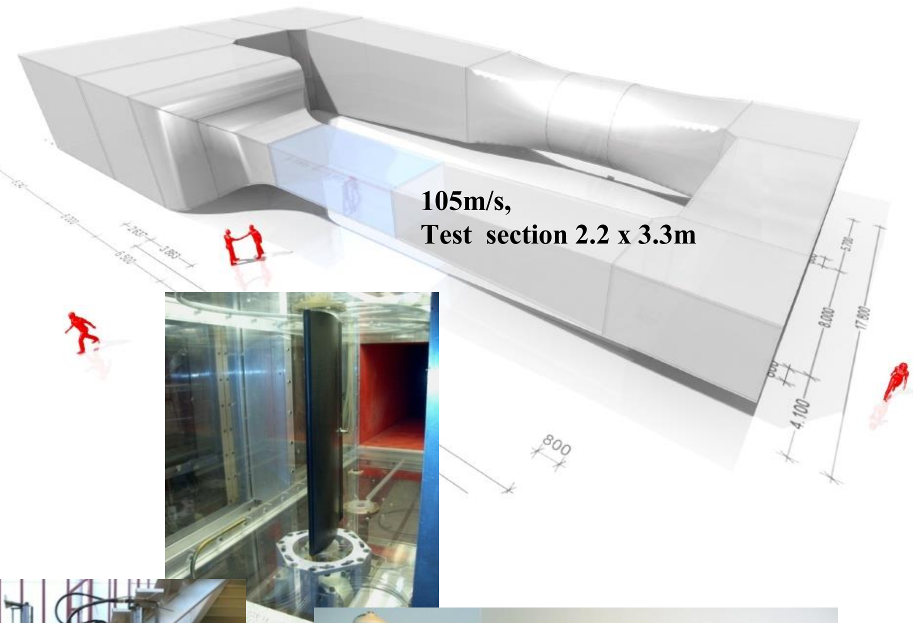
Society
- Economics
- Environmetal aspects
- Integration with other technologies
- Storage issues

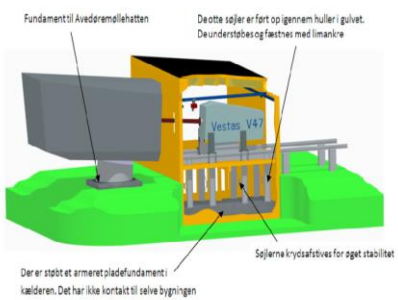
Wind Power Systems
- Grid integration
- Wake interaction
- Layout of wind farms
- Maintenance

Activities at DTU Wind Energy

Research and test facilities







Research Facilities

Existing:

- Wind turbines at Risø Campus for research and courses;
- Test Station for Large Wind Turbines at Høvsøre;
- Risø met-mast
- Blade test Facility for Research;
- 1 MW drive-train test facility ;
- Measurement stations and equipment, incl. Lidars;
- PC-clusters;
- Structural test laboratory;
- Material tests lab, incl. Microscopes etc
- Fiber lab
- Smaller wind tunnels; and
- The WindScanner facility.

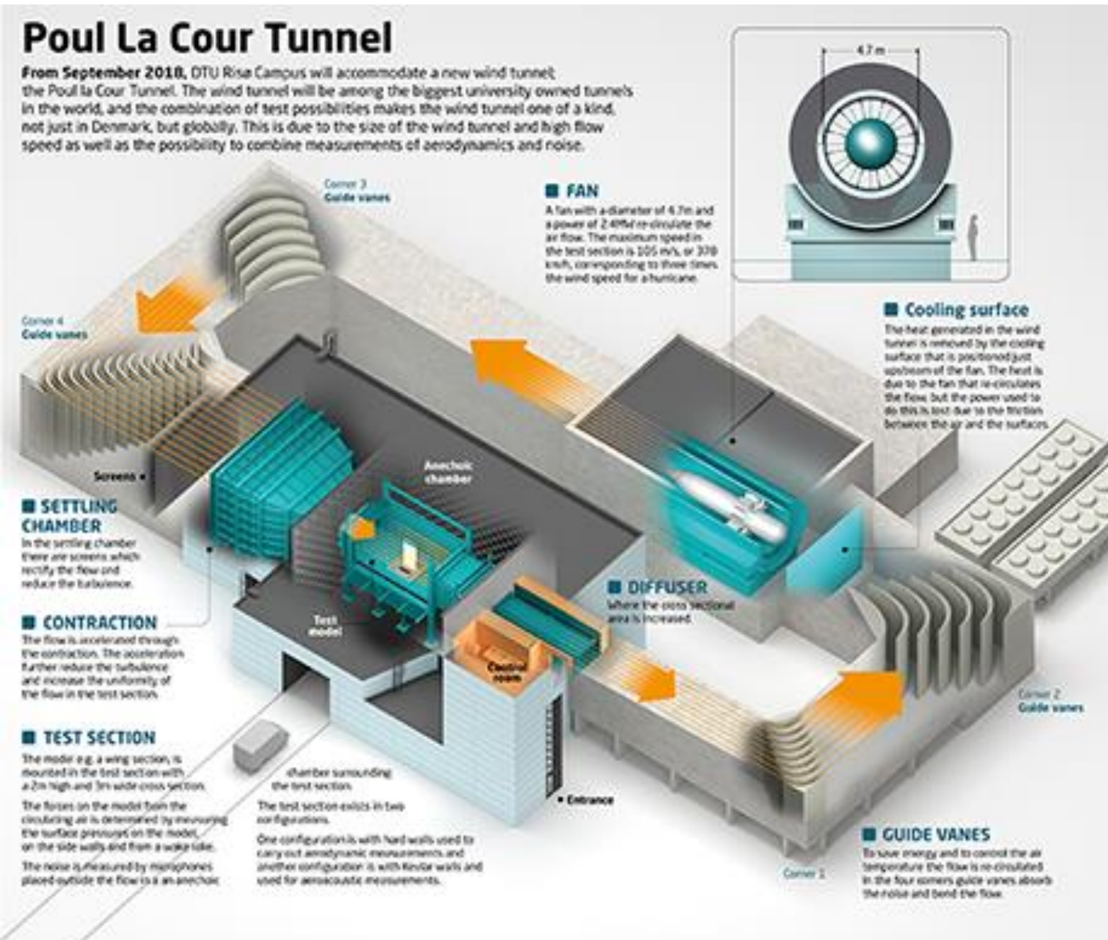
Under development or in planning phase:

- Test Station for ”Very” Large Wind Turbines at Østerild;
- Østerild Grid Test Facility;
- National Wind Tunnel
- 10 10/20 MW drive-train test facility in cooperation with LORC

Wind turbine blade testing

Commercial testing at Blade Test centre A/S, a private limited company with the following shareholders:
Det Norske Veritas AS
Technical University of Denmark
FORCE Technology

New wind tunnel for blade testing



Risø Test Stations – Prototype Testing

5 test beds

< 165 m

< 8 MW

Spacing 300 m

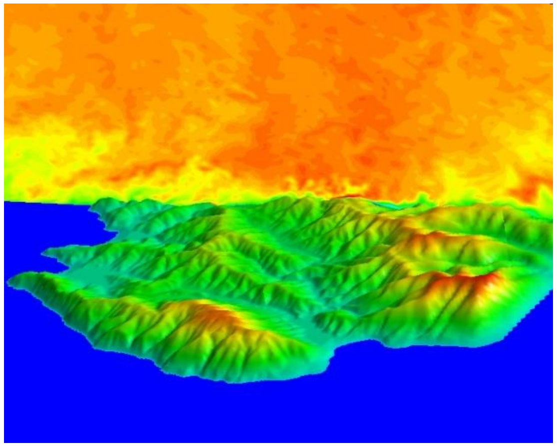
7 test beds

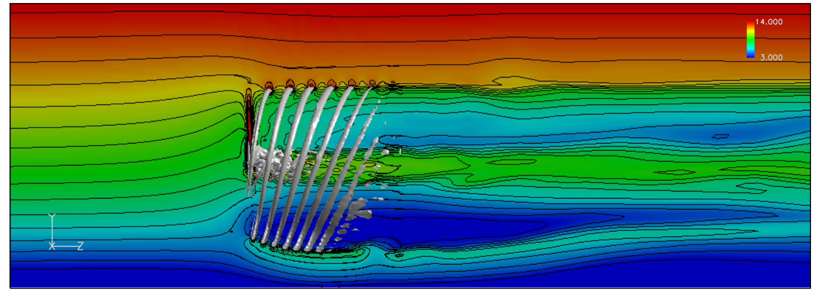
< 250 m

< 16 MW

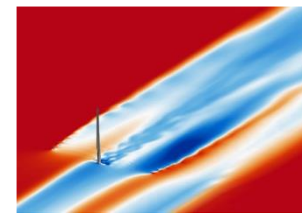
Spacing 600 m

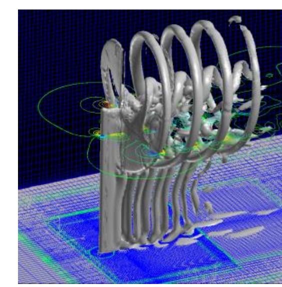
Advanced Wind Turbine Aerodynamics -modeling and exp. validation

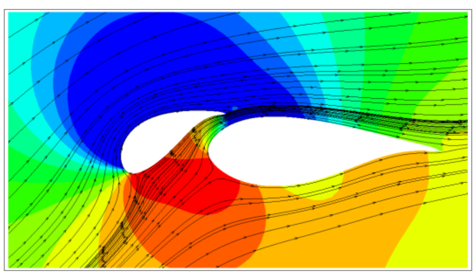


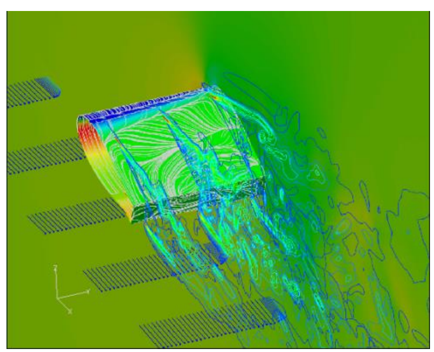


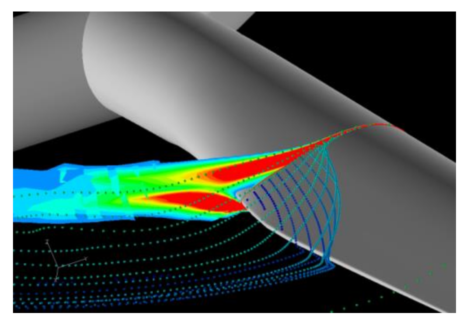
Exp. validation





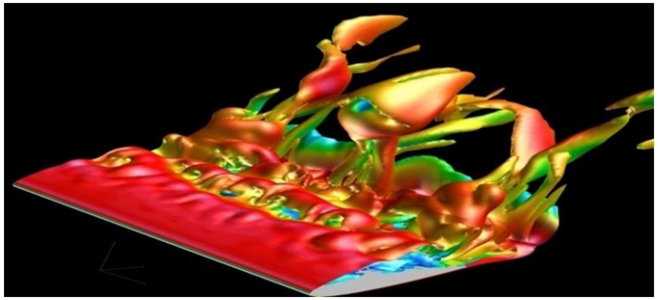




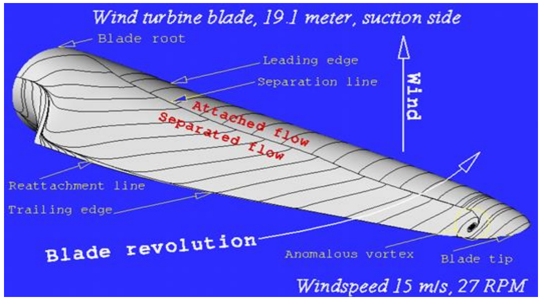


Main Subjects in Wind Turbine Aerodynamics

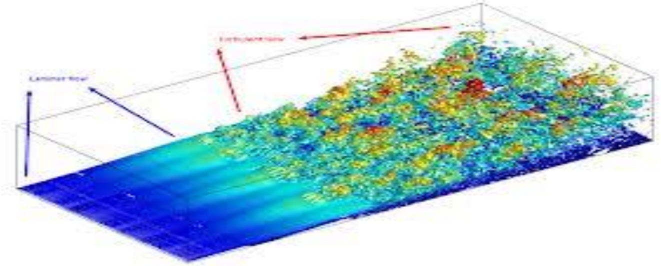
Airfoils and boundary layers



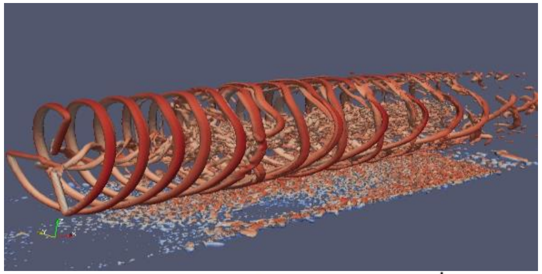
Rotor Aerodynamics



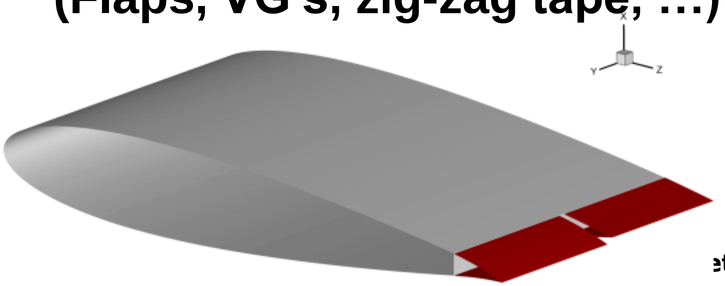
Laminar/Turbulent Transition



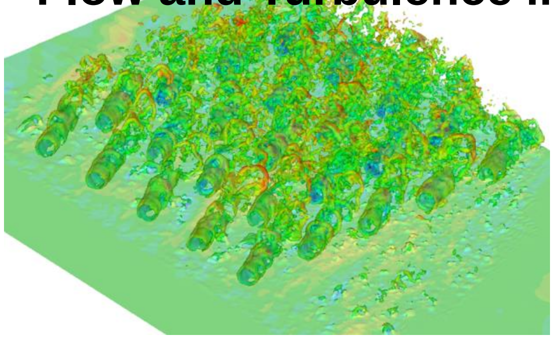
Wake Interaction



Aerodynamic Accessories
(Flaps, VG's, zig-zag tape, …)



Flow and Turbulence in Wind Farms



The Scale Challenge in Wind Energy

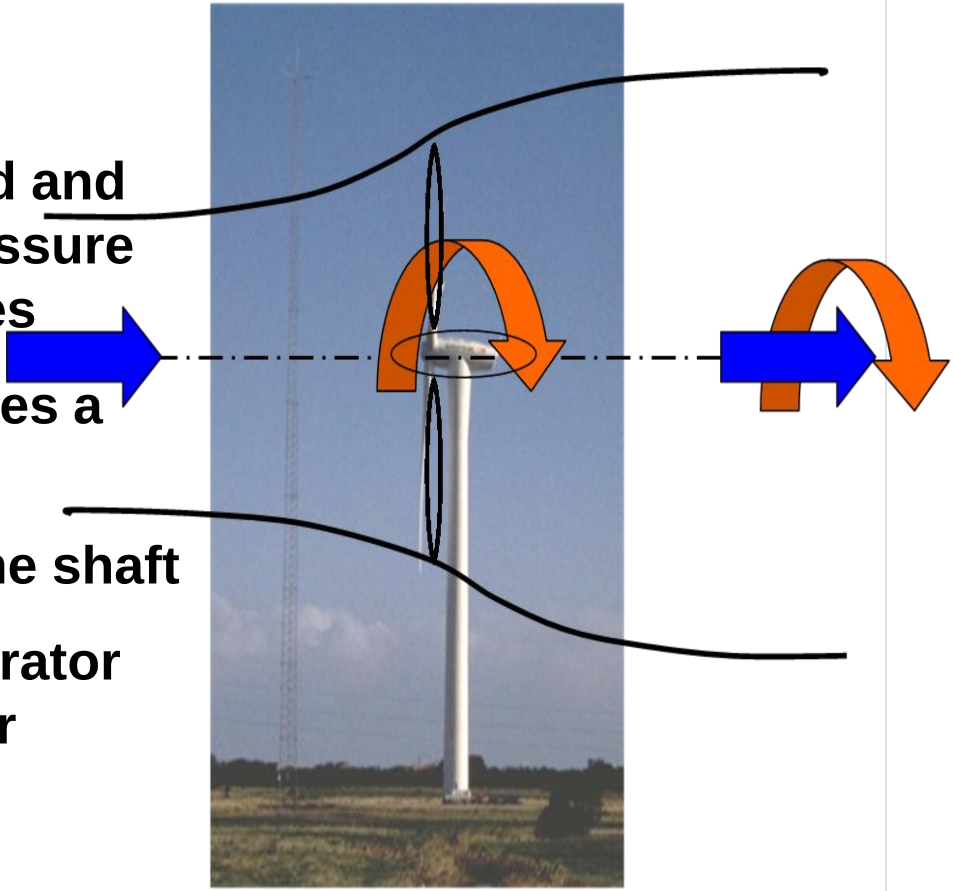
Turbulent scales:

	Length scale (m)	Velocity scale (m/s)	Time scale (s)
Airfoil boundary layer	10^{-3}	10^{2}	10^{-5}
Airfoil	1	10^{2}	10^{-2}
Rotor	10^{2}	10	10
Cluster	10^{3}	10	10^{2}
Wind farm	10^{4}	10	10^{3}

Wind Energy

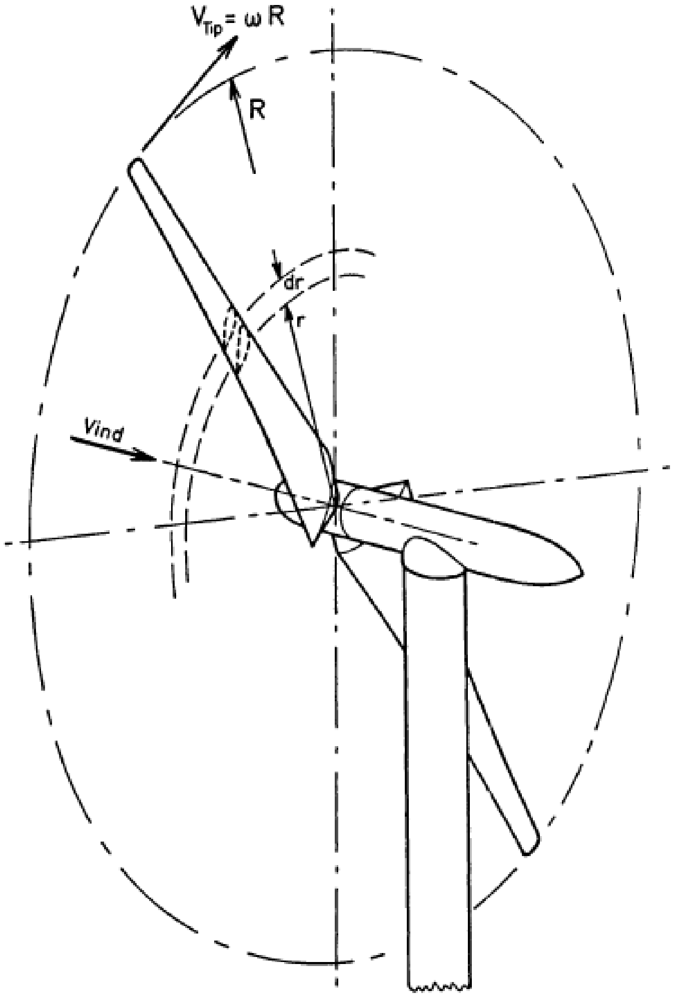
How does a wind turbine work?

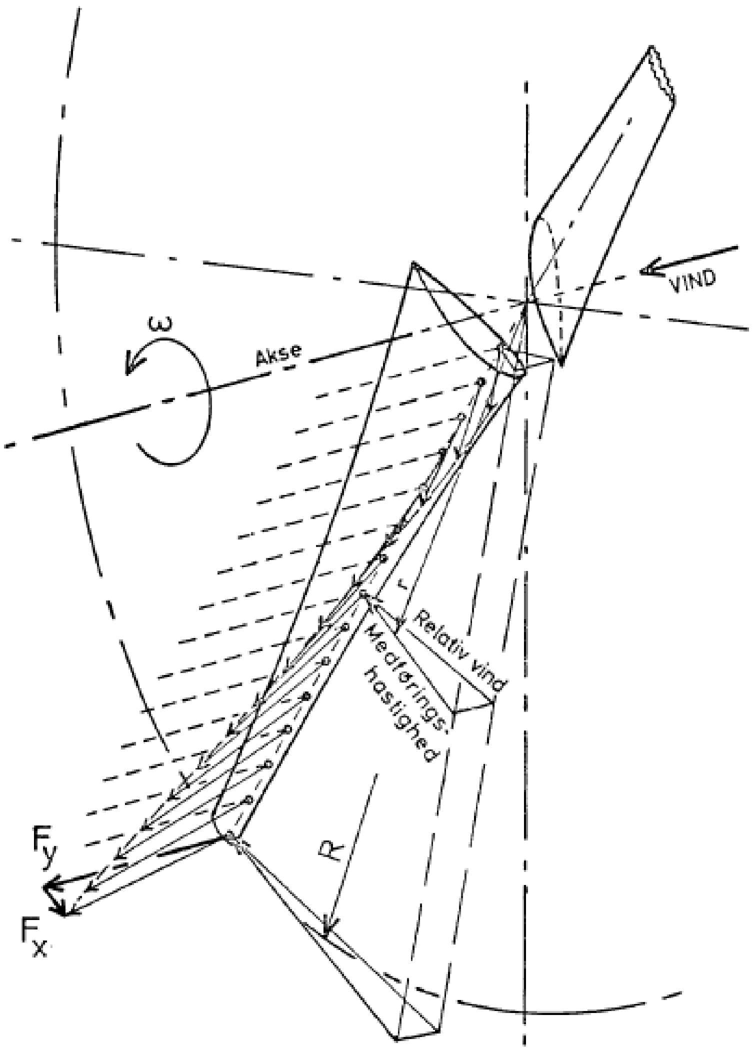
- The wind hits the rotor plane
- The combinition of wind speed and blade rotation results in a pressure distribution on the rotor blades
- The pressure distribution causes a turning moment (torque)
- The turning moment rotates the shaft
- The shaft is coupled to a generator that produces electrical power



Wind Energy

Aerodynamic forces:





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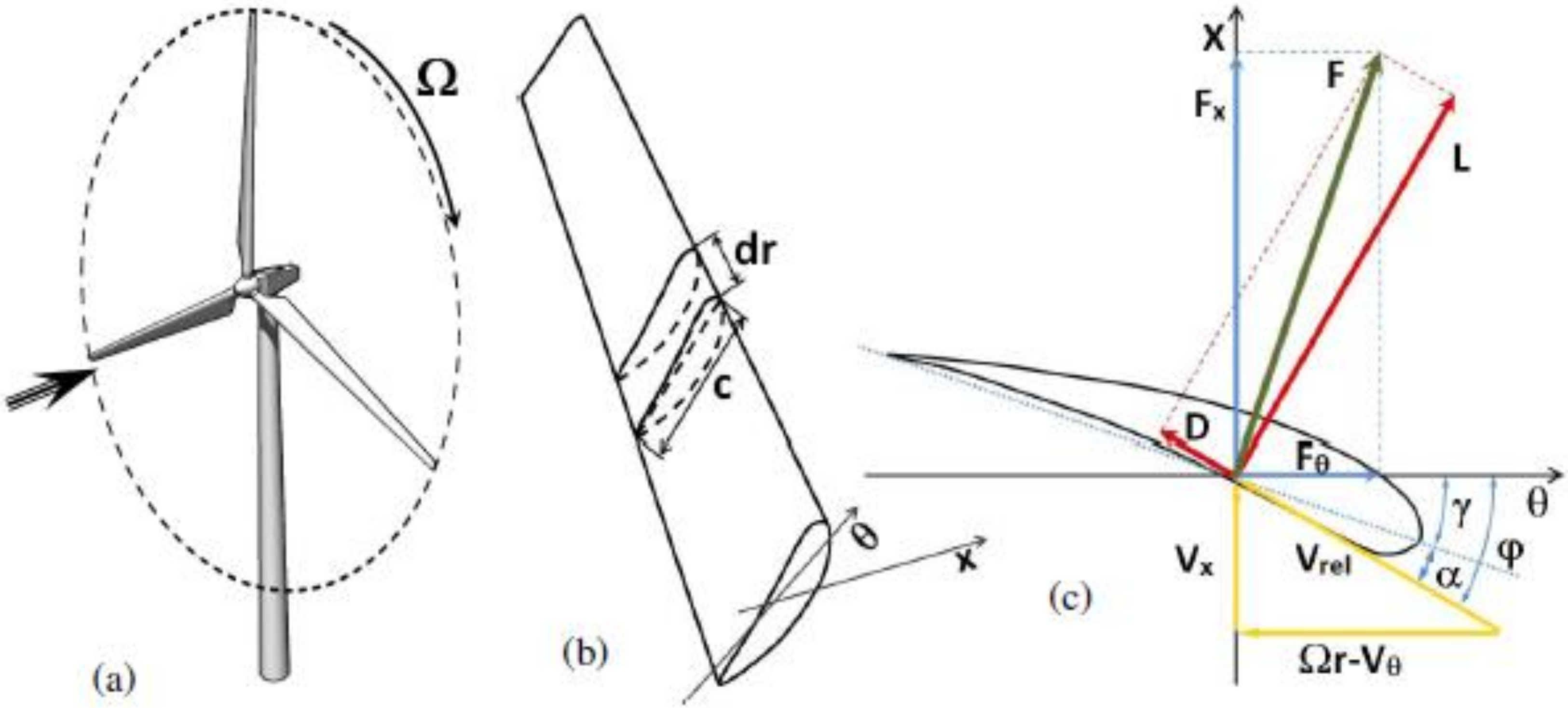
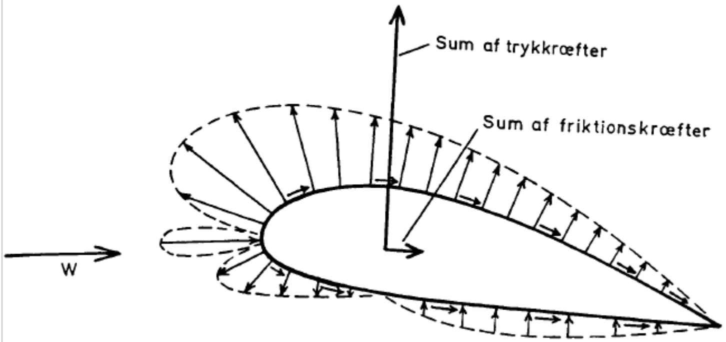
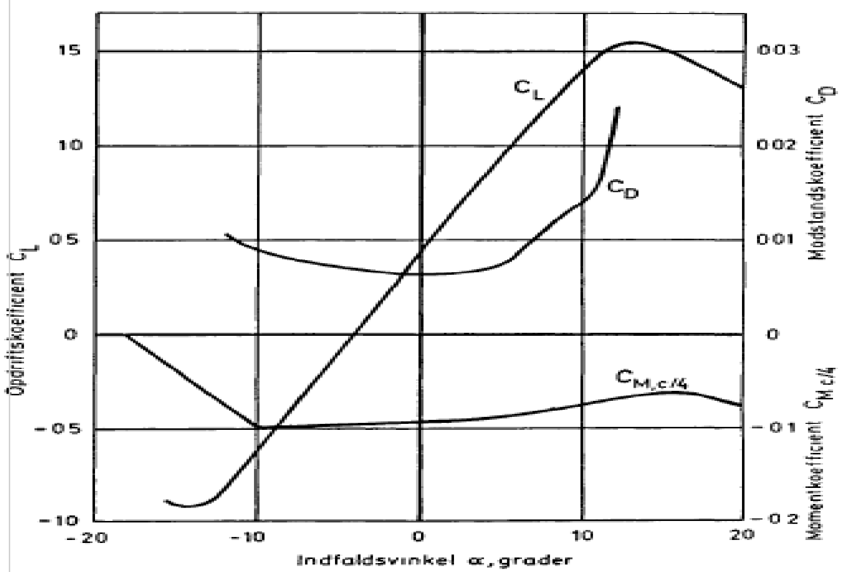
Aerodynamic forces and geometry :

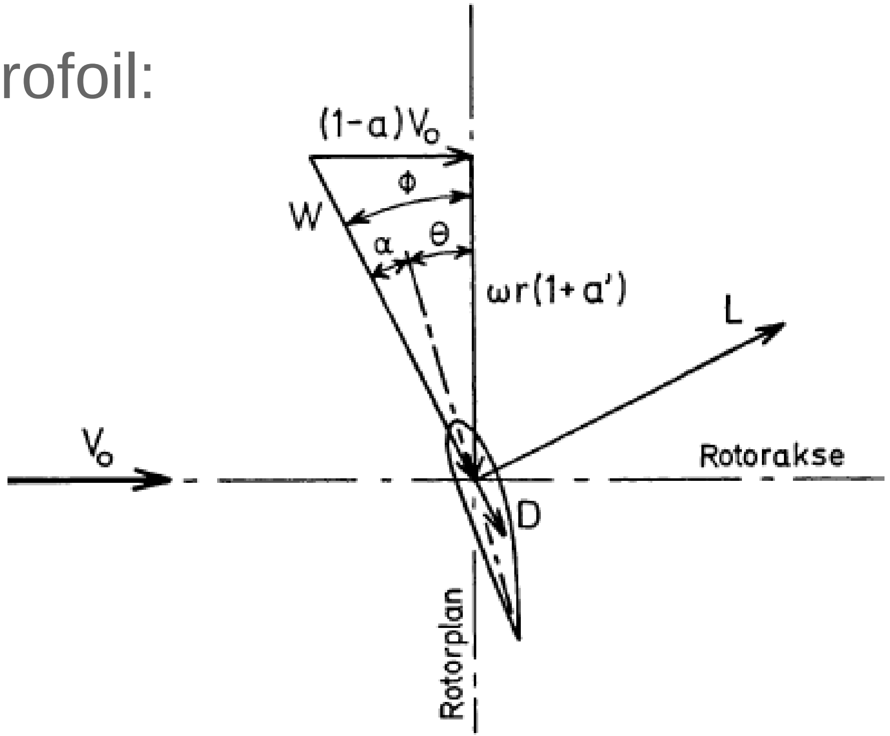


Wind Energy

Pressure and forces on aerofoil:







$$C_y = C_L \cos\Phi + C_D \sin\Phi$$

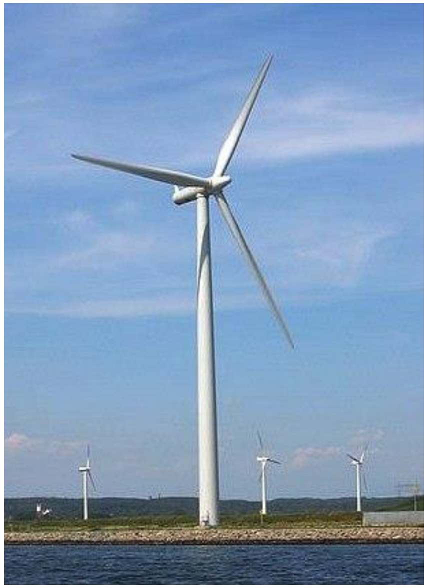
$$C_x = C_L \sin\Phi - C_D \cos\Phi$$

The Optimum Rotor

- What is the optimum number of blades ?
- What is the optimum operating condition (TSR)?
- What is the maximum efficiency?

Wind Turbine Rotor Aerodynamics

What is the optimum number of blades?



3 blades ?

Wind Turbine Rotor Aerodynamics

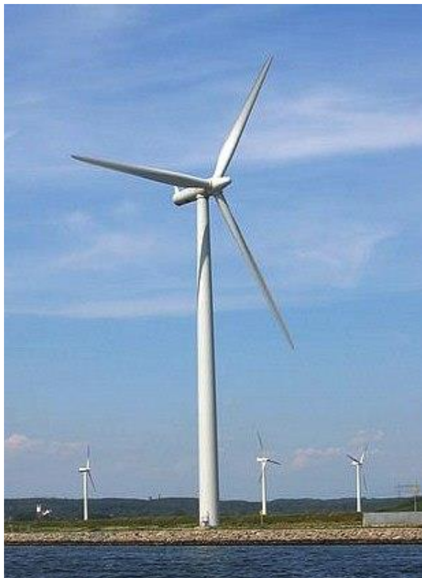
What is the optimum number of blades?

3 blades ?

many blades ?

Wind Turbine Rotor Aerodynamics

What is the optimum number of blades?



3 blades ?

many blades ?

2 blades ?

Wind Turbine Rotor Aerodynamics

What is the optimum number of blades?

3 blades ?

many blades ?

2 blades ?

1 blade ?

Why 3 blades?

- **Aerodynamics:** **Close to optimum**
- **Structural-dynamics:** **No gyroscopic forces in yaw**
- **Estetics:** **Harmonic rotation**
- **Historics:** **Tradition of the ’Danish Concept’**

Optimum rotor with infinite number of blades

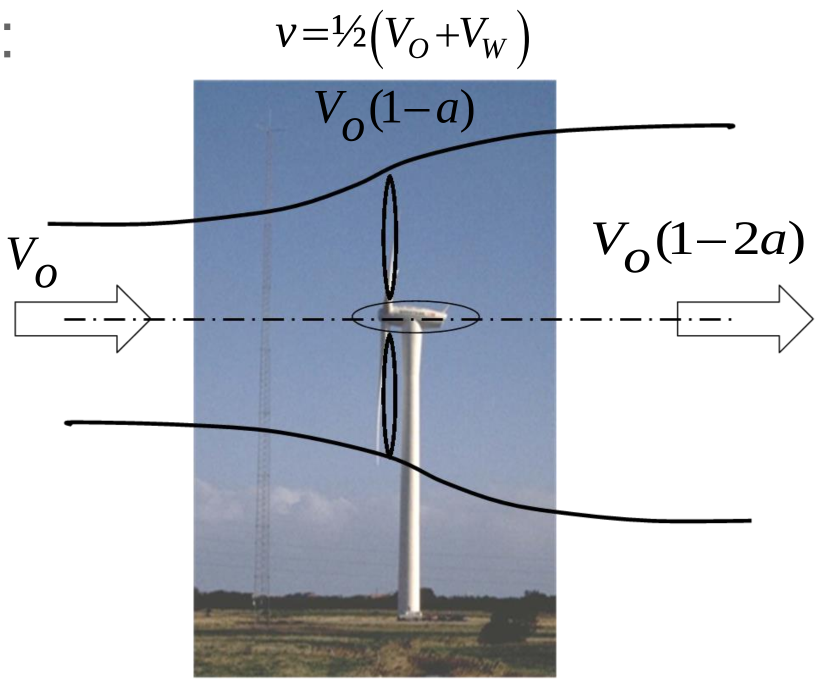
1-D Axial momentum theory:

Axial interference factor: $a = 1 - v/V_o$

Thrust Coefficient: $C_T = \dfrac{T}{\tfrac{1}{2}\rho A_o V_o^2}$

Power coefficient: $C_P = \dfrac{P}{\tfrac{1}{2}\rho A_o V_o^3}$

$$\boxed{\begin{array}{l} T = \dot{m}\cdot\Delta V \Rightarrow T = \rho A_o v\cdot(V_o - V_W) \\ P = vT \Rightarrow \quad P = \rho A_o v^2 (V_o - V_W) \end{array}}$$



$$\boxed{\begin{array}{c} C_T = 4a(1-a) \\ C_P = 4a(1-a)^2 \end{array}}$$

Betz limit:

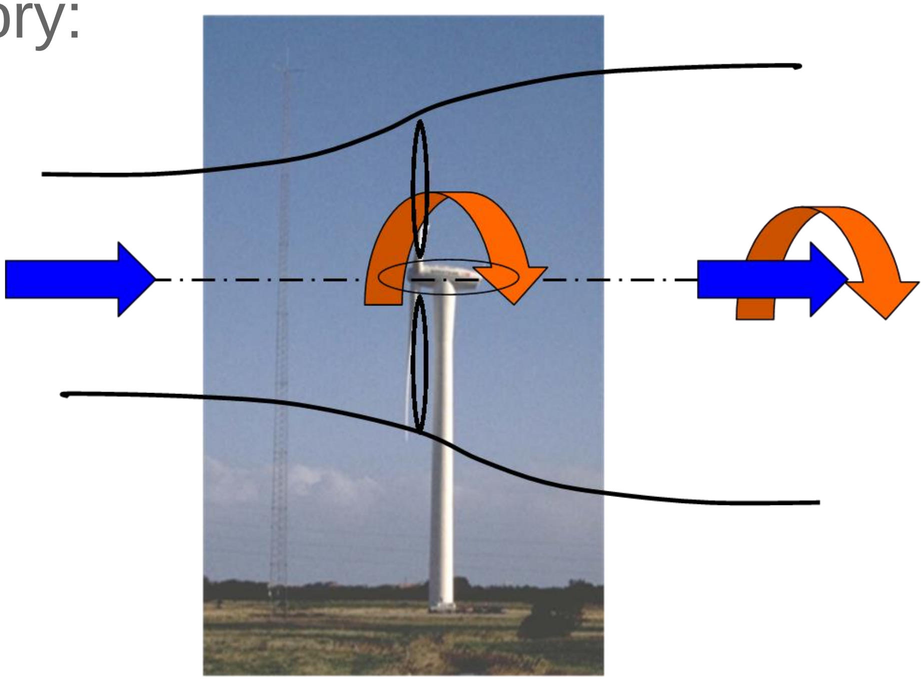
$$\boxed{a = \frac{1}{3}: \quad C_{P\max} = \frac{16}{27} = 0.593}$$

Optimum rotor with infinite number of blades

Generel momentum theory:

$$a = 1 - v_o / V$$

$$a' = \frac{u_{\theta o}}{\Omega_o r}$$



Euler's turbine equation:

$$C_P = 8\lambda^2 \int_0^1 a'(1-a)x^3 dx$$

TSR	$C_{P\max}$
0.5	**0.288**
1.0	**0.416**
1.5	**0.480**
2.0	**0.512**
2.5	**0.532**
5.0	**0.570**
7.5	**0.582**
10.0	**0.593**

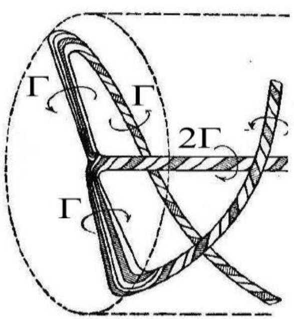
Condition for optimum operation:

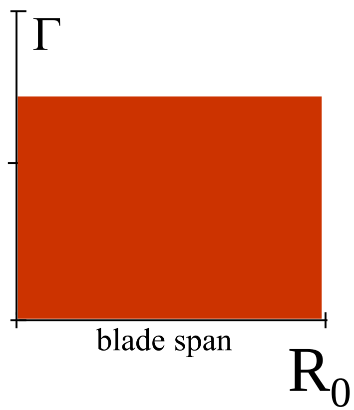
$$a' = (1-3a)/(4a-1)$$

Two definitions of the ideal rotor

Joukowsky (1912)

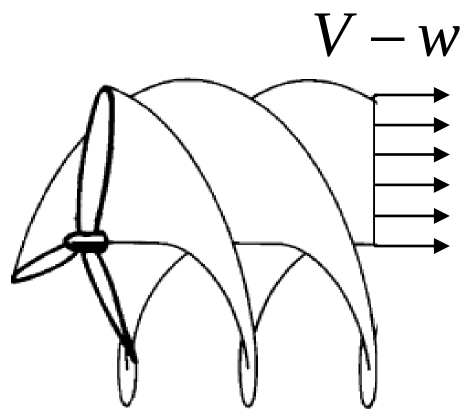
$$\Gamma = const$$

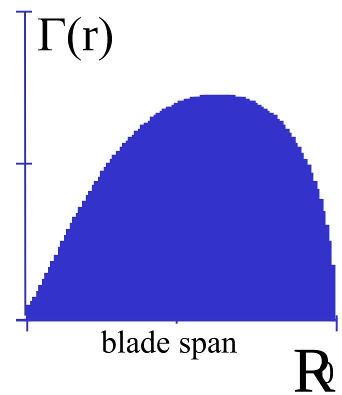




Betz (1919)

$$V - w = const$$

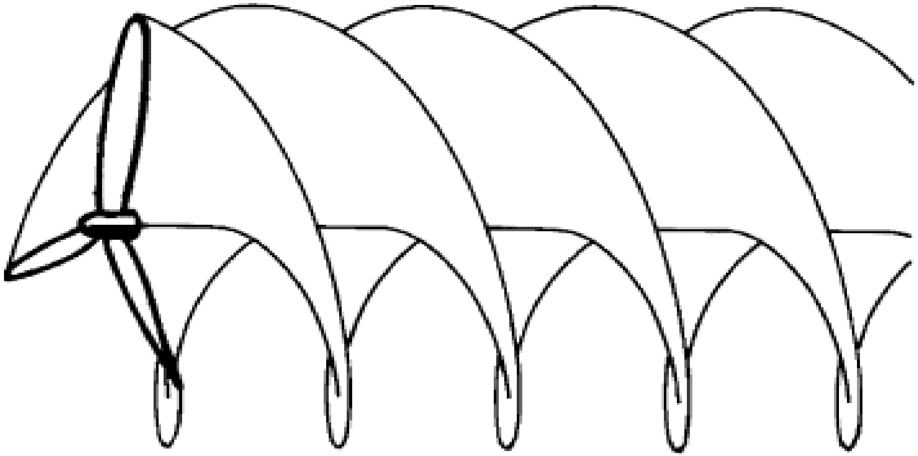




In both cases only conceptual ideas were outlined for rotors with finite number of blades, whereas later theoretical works mainly were devoted to rotors with <u>infinite</u> blades!

Betz’ condition for maximum efficiency of a rotor with a finite number of blades

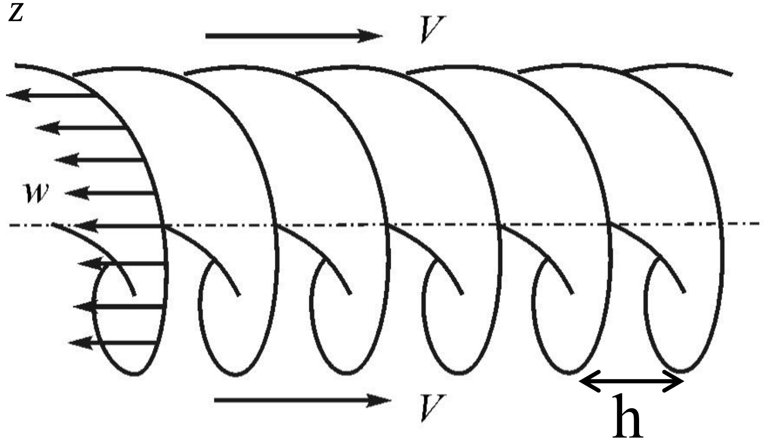
Maximum efficiency is obtained when the pitch of the trailing vortices is constant and each trailing vortex sheet translates backward as an undeformed regular helicoidal surface



The geometry of the wake:

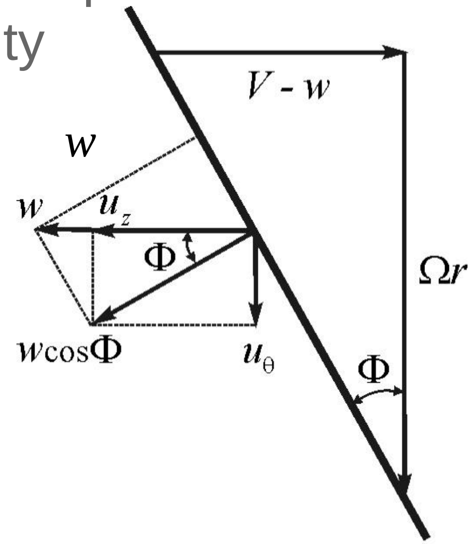
Axial induced velocity:

$$u_z = w\cos^2\Phi$$



a

Movement of vortex sheet with constant pitch and constant velocity



b

Tangential induced velocity:

$$u_\theta = w\cos\Phi\sin\Phi$$

Pitch:

$$h = 2\pi r\tan\Phi / B = 2\pi(V - w)/B\Omega$$

$$\tan\Phi = \frac{dz}{rd\theta} = (V - w)/\Omega r$$

Induced velocities (Okulov, 2007):

$$u_r \cong -\frac{\gamma}{2\pi r}\frac{\sqrt[4]{\left(l^2+r^2\right)\left(l^2+r_0^2\right)}}{l}\,\mathrm{Im}\left[\frac{e^{i\chi}}{e^{\mp\xi}-e^{i\chi}} \pm \frac{l}{24}\left(\frac{2l^2+9r_0^2}{\left(l^2+r_0^2\right)^{3/2}} - \frac{2l^2+9r^2}{\left(l^2+r^2\right)^{3/2}}\right)\ln\left(1-e^{\pm\xi+i\chi}\right)\right]$$

$$u_z \cong \frac{\gamma}{2\pi l}\begin{Bmatrix}1\\0\end{Bmatrix} + \frac{\gamma}{2\pi l}\frac{\sqrt[4]{l^2+r_0^2}}{\sqrt[4]{l^2+r^2}}\,\mathrm{Re}\left[\frac{\pm e^{i\chi}}{e^{\mp\xi}-e^{i\chi}} + \frac{l}{24}\left(\frac{3r^2-2l^2}{\left(l^2+r^2\right)^{3/2}} + \frac{9r_0^2+2l^2}{\left(l^2+r_0^2\right)^{3/2}}\right)\ln\left(1-e^{\xi+i\chi}\right)\right]$$

$$u_\theta = \left(u_0 - u_z\right) l/r \qquad l = h/2\pi R = (r/R)\tan\Phi$$

$$\chi = \theta - z/l$$

$$u_0 = \gamma/2\pi l$$

$$e^{\xi} = \frac{r'}{r_0'} = \frac{r}{r_0}\frac{\left(1+\sqrt{1+r_0^2/l^2}\right)\exp\left(\sqrt{1+r^2/l^2}\right)}{\left(1+\sqrt{1+r^2/l^2}\right)\exp\left(\sqrt{1+r_0^2/l^2}\right)}$$

Optimum circulation distribution

Goldstein function: $G(r) = \Gamma(r)/hw = B\Gamma\Omega / 2\pi w(V - \frac{1}{2}w)$

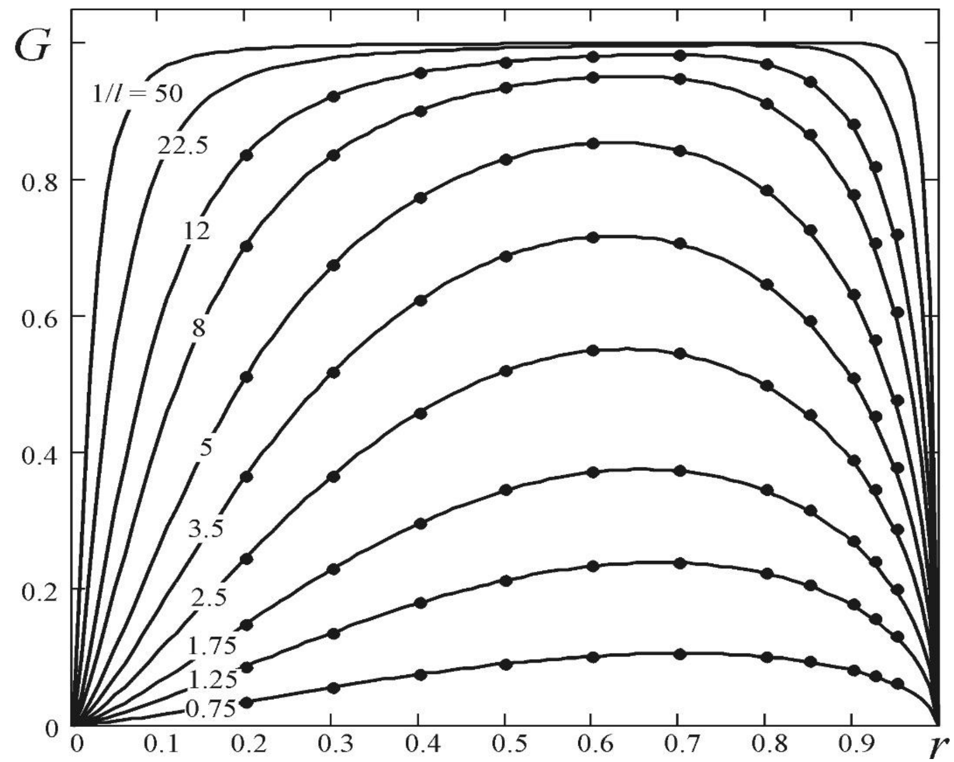
Obtained by solving the matrix equation:

$$\left[\underline{\underline{A}}\right]\underline{\gamma} = \frac{x_i^2}{l^2 + x_i^2}$$

where

$$\underline{\gamma} = \left[\gamma_1, \gamma_2, \ldots, \gamma_N\right]^T$$

$$\gamma_i = \Gamma_i / w$$



Optimum lift distribution:

Goldstein function: $G(r) = \Gamma(r)/hw = B\Gamma\Omega / 2\pi w(V - ½w)$

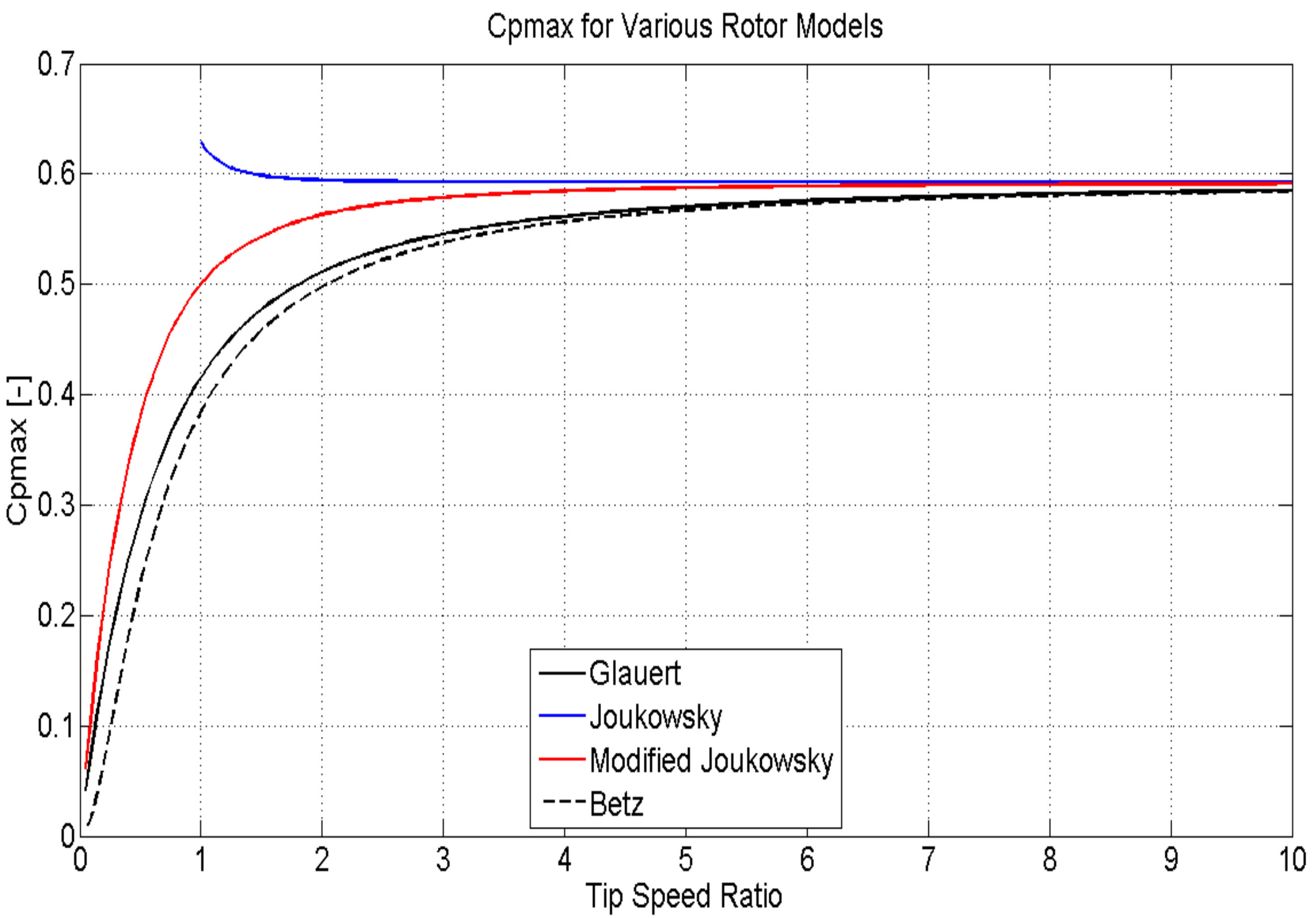
Kutta-Joukowski theorem: $\Gamma = ½cC_L U_o \Rightarrow cC_L = \dfrac{2\Gamma}{U_o}$

Combining these equations, we get

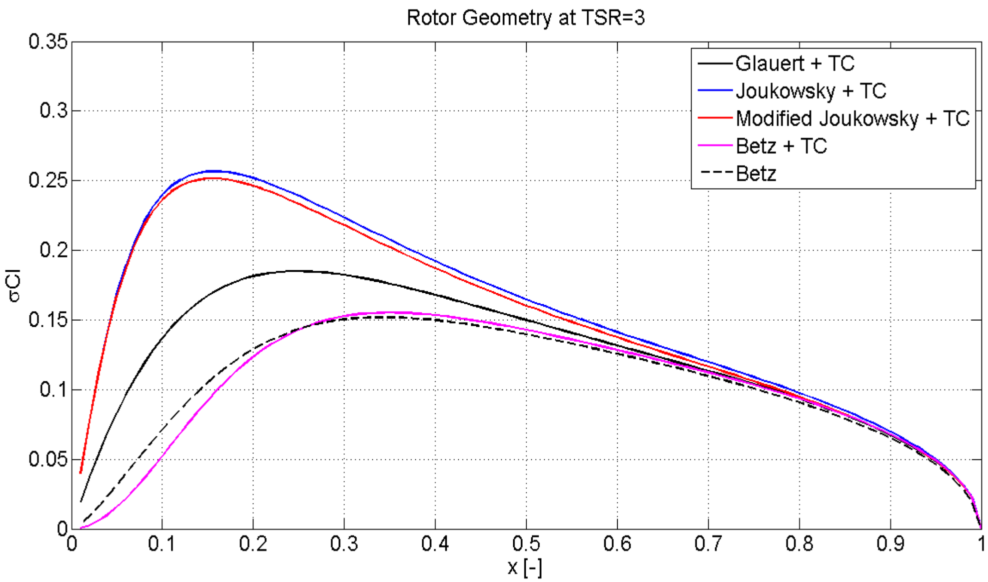
$$\boxed{\sigma C_L = \frac{2\overline{w}(1-½\overline{w})G(r/R)}{\lambda(U_o/V)}}$$

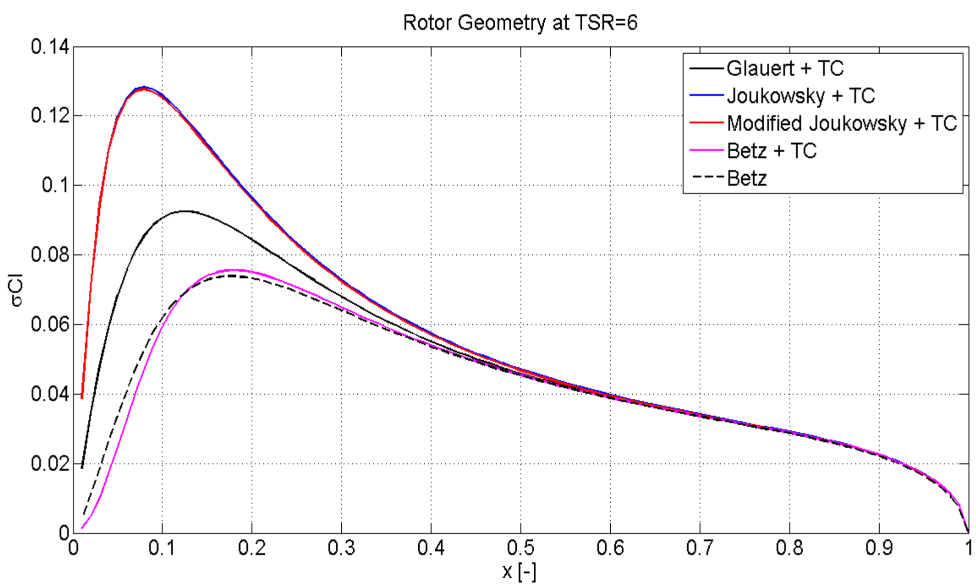
Solidity: $\sigma = \dfrac{Bc}{2\pi R_o}$ Tip Speed ratio: $\lambda = \dfrac{\Omega R}{V}$

Optimum Power Coefficients

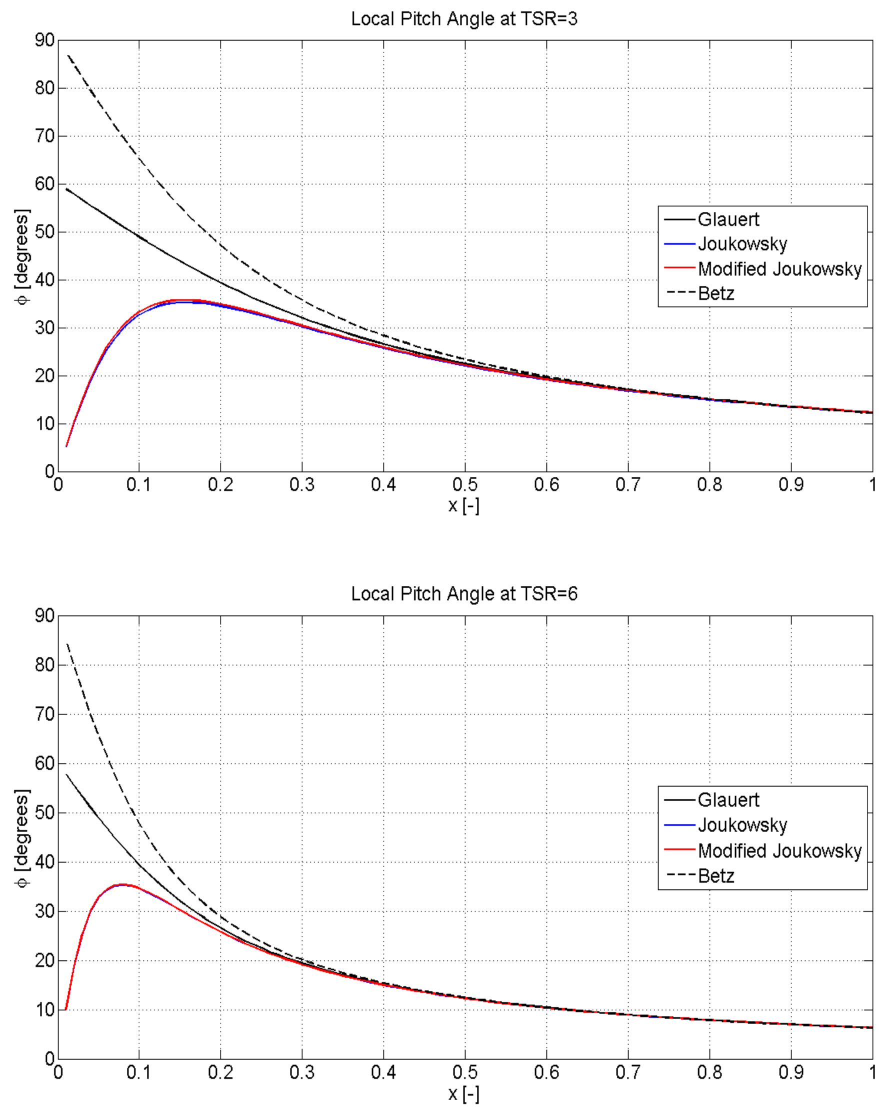


Optimum rotor: planforms

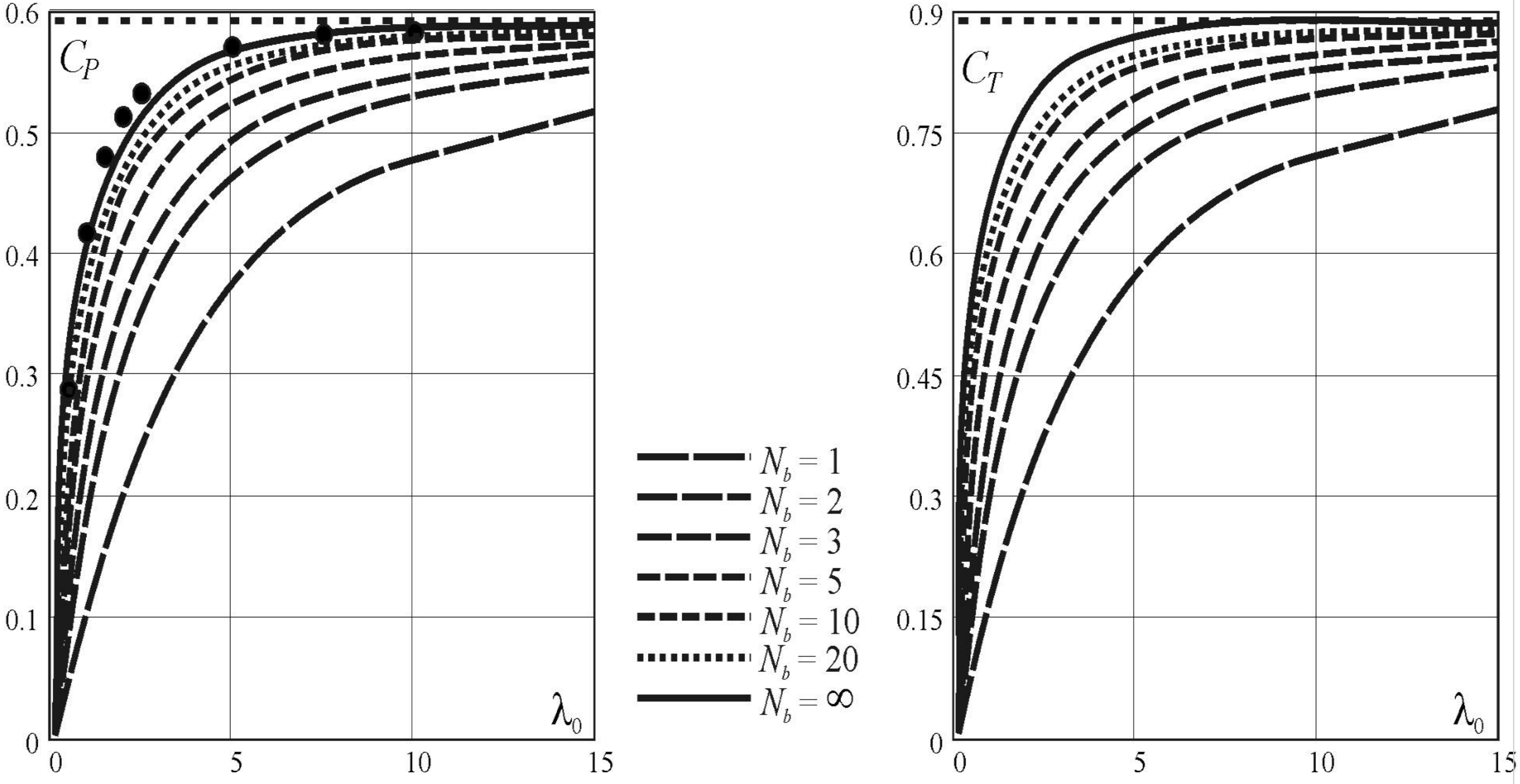




Optimum rotor: twist distributions

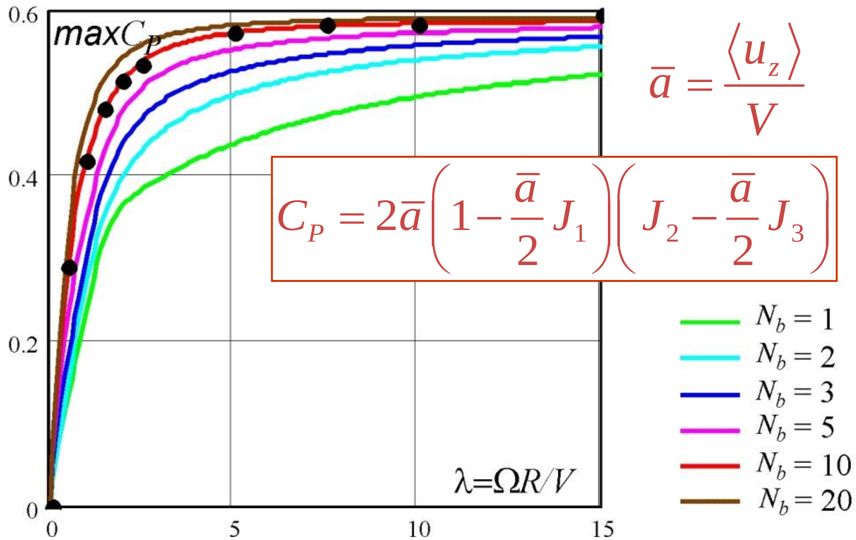


The optimum rotor (Betz theory):



Comparison of maximum power coefficients

Solution of Joukowsky rotor (Okulov & Sørensen, 2010)



$$\bar{a} = \frac{\langle u_z \rangle}{V}$$

$$\boxed{C_P = 2\bar{a}\left(1 - \frac{\bar{a}}{2} J_1\right)\left(J_2 - \frac{\bar{a}}{2} J_3\right)}$$

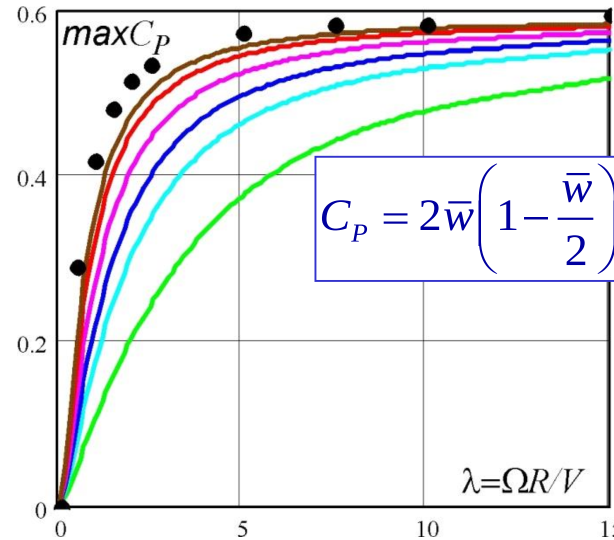
$$\bar{a} = \frac{J_1 J_2 + J_3 - \sqrt{J_1^2 J_2^2 - J_1 J_2 J_3 + J_3^2}}{3 J_1 J_3}$$

$$J_1 = 1 + 2\frac{l^2}{R^2}$$

$$J_2 = 1 + \frac{1}{2}\frac{\varepsilon}{R} + \frac{1}{6}\frac{\varepsilon^2}{R^2}$$

$$J_3 = 2l\int_0^1 \frac{\tilde{u}_z(x)}{a} x dx$$

Solution of Betz rotor (Okulov&Sørensen, 2008)



$$\bar{w} = \frac{w}{V}$$

$$\boxed{C_P = 2\bar{w}\left(1 - \frac{\bar{w}}{2}\right)\left(I_1 - \frac{\bar{w}}{2} I_3\right)}$$

Difference between power coefficients

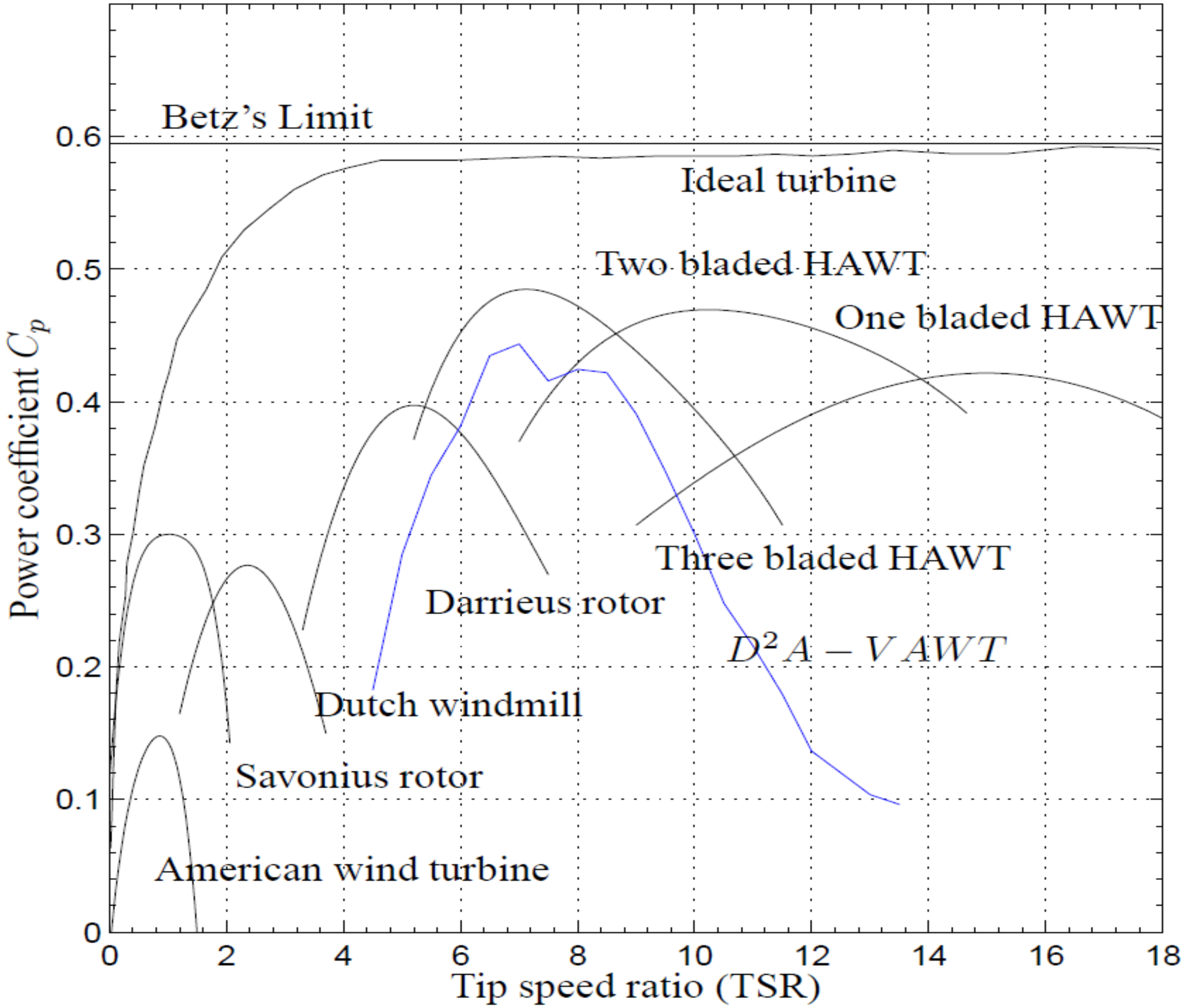
$$\bar{w} = \frac{2\left(I_1 + I_3 - \sqrt{I_1^2 - I_1 I_3 + I_3^2}\right)}{3 I_3}$$

Mass coefficient $I_1 = 2\int_0^1 G(x) x dx$

"Axial loss factor" $I_3 = 2\int_0^1 G(x) \frac{x^3 dx}{x^2 + l^2}$

Wind Energy

Wind turbine performance :



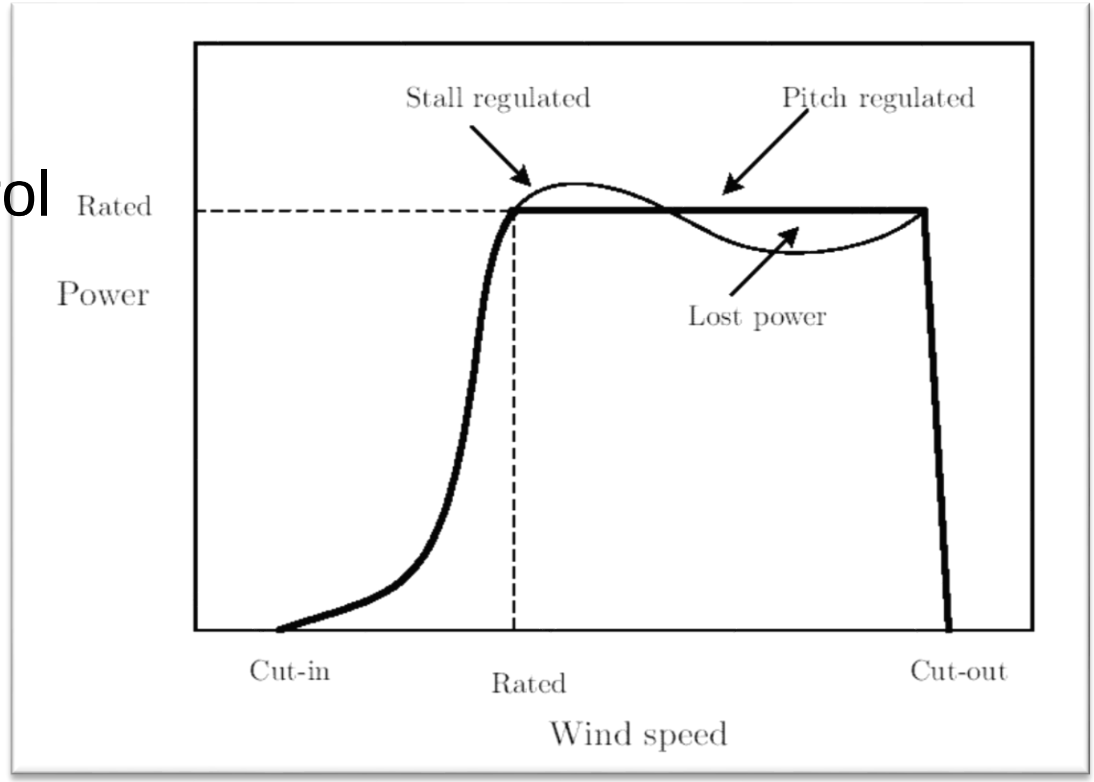
Control and regulation of wind turbines

Stall control

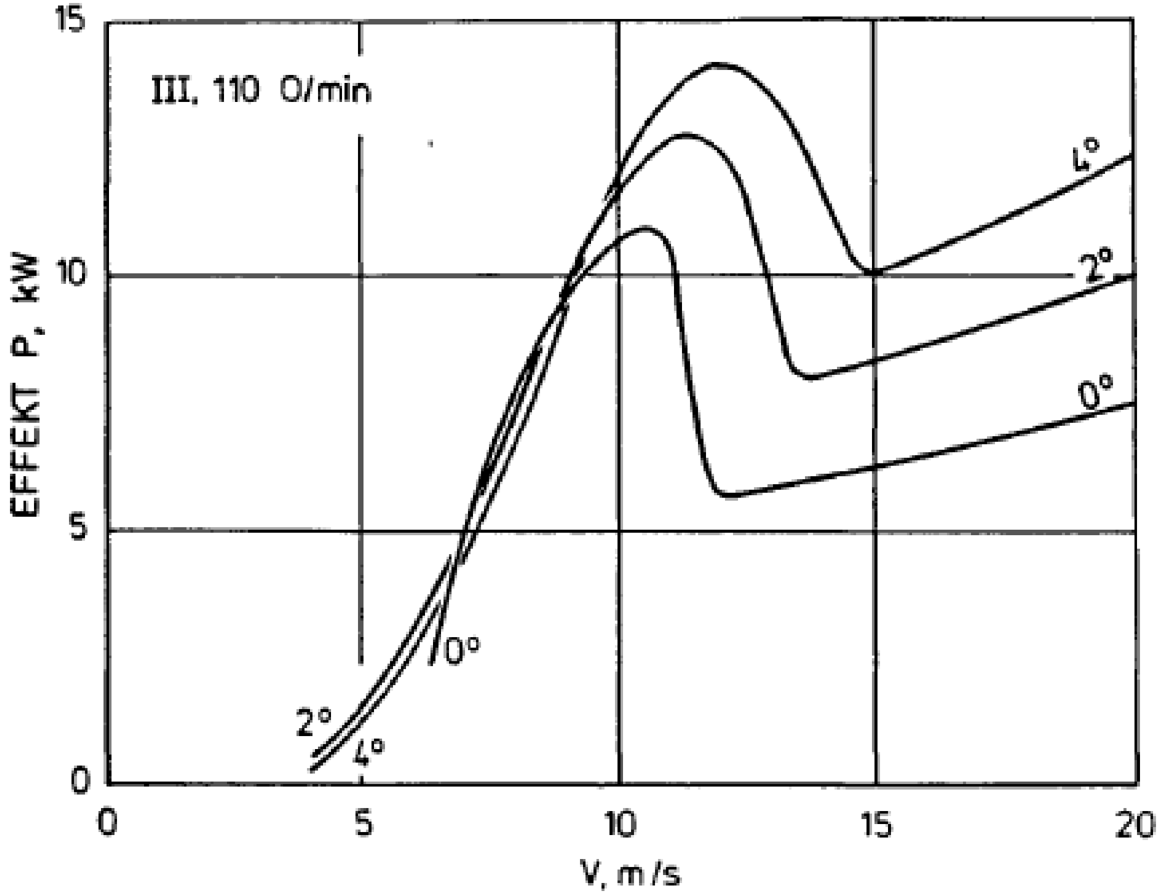
Active Stall control

Pitch control

Variable speed



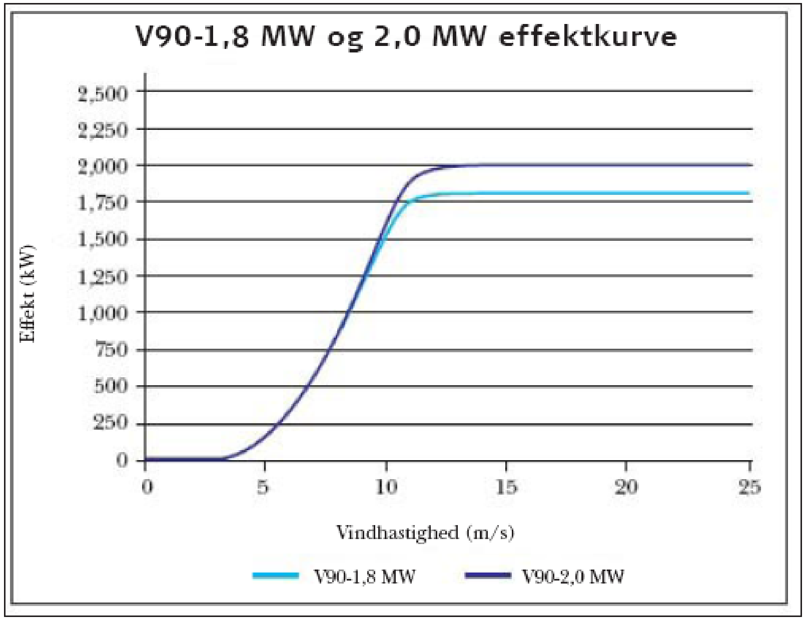
Stall-regulated wind turbine: Computed power curve

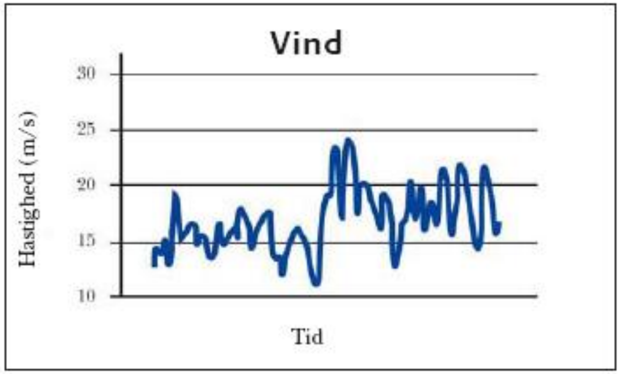


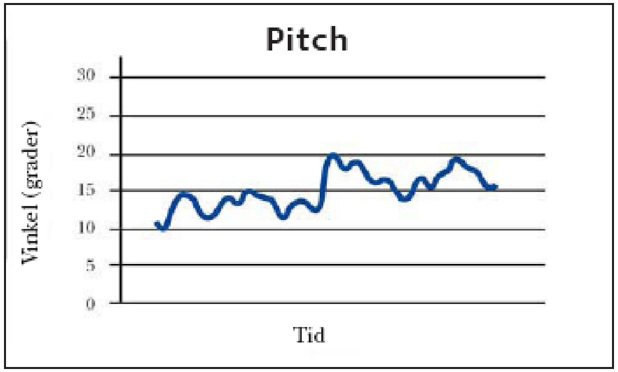
Wind Turbine

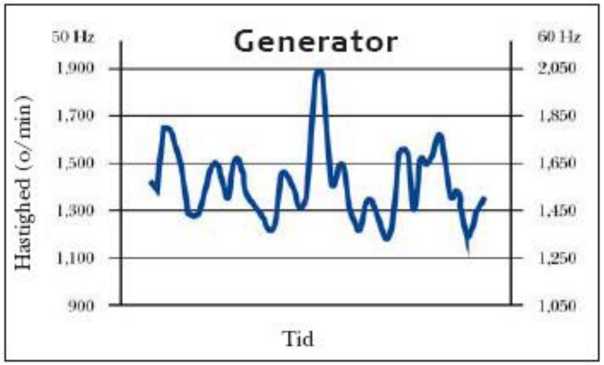
Modern Wind Turbine :

- **Pitch-regulated**
- **P=2 MW; D=90 m**
- **Nom. Tip speed.: 70 m/s**
- **Rotor: 38t, Nacelle: 68t; Tower: 150t**
- **Control: OptiSpeed; OptiTip**









Wind Turbine Aerodynamics

Need for models capable of coping with:

- **Dynamic simulations of large deformed rotors**
- **Complex geometries: Rotor tower interaction**
- **Adjustable trailing edge flaps**
- **Various aerodynamic accessories, such as vortex generators, blowing, Gourney flaps and roughness tape**
- **3-dimensional stall including laminar-turbulent transition**
- **Unsteady, three-dimensional and turbulent inflow**
- **Interaction between rotors and terrain**
- **Complex terrain and wind power meteorology**
- **Offshore wind energy: Combined wind and wave loadings**

Blade Element Momentum Model

Basic ingredients of the BEM model:

- **Based on 1-D momentum theory assuming annular independency**
- **Loading computed using tabulated static airfoil data**
- **Dynamic stall handled through ’dynamic stall’ models**
- **3-dimensional stall introduced through modifications**
- **Tip Flows based on (Prandtl) tip correction**
- **Yaw treated through simple modifications**
- **Heavily loaded rotors treated through Glauert’s approximation**
- **Wakes and park effects modelled using axisymmetric momentum theory**

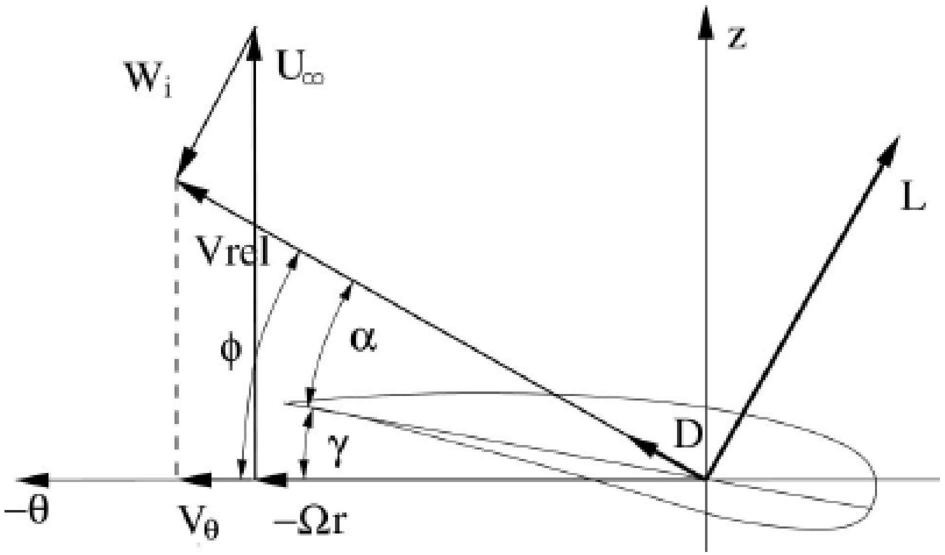
Blade Element Momentum Model

Advantages of using the BEM model:

- Extremely fast on a PC
- Can in principle cope with all flow situations
- Easy to couple with an aeroelastic code, such as Flex
- Easy to couple with turbulent inflow model
- Many years of experience in using the model
- Performs very well at design conditions
- Capable of delivering results at off-design conditions

The BEM model is today the industrial standard used by all producers of wind turbines and wind turbine blades

Blade Element Momentum Model



Blade element

$$\frac{dT}{dr} = BF_n = \frac{1}{2}\rho c B V_{rel}^2 \cdot C_n,$$

$$\frac{dM}{dr} = BrF_t = \frac{1}{2}\rho c B r V_{rel}^2 \cdot C_t$$

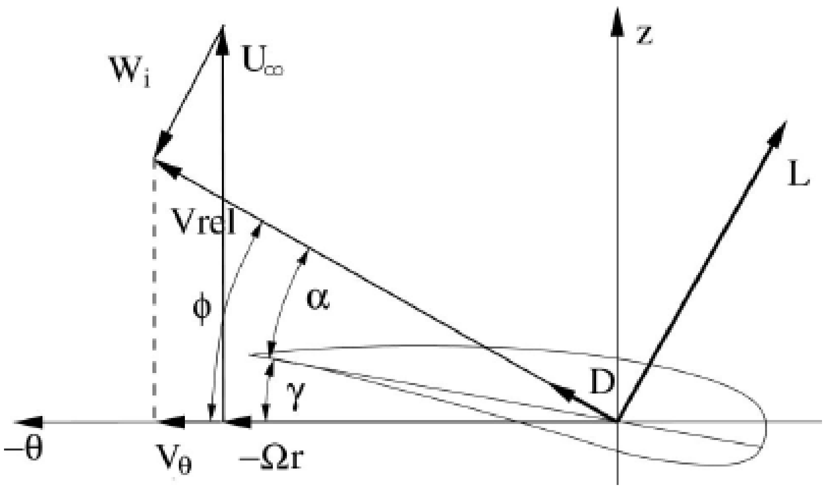
$$C_n = C_l \cos\phi + C_d \sin\phi$$

$$C_t = C_l \sin\phi - C_d \cos\phi$$

Airfoil data: $L = \frac{1}{2}\rho V_{rel}^2 c C_L$

$$D = \frac{1}{2}\rho V_{rel}^2 c C_D$$

Blade Element Momentum Model



Induced velocity:

$$W_i = (-aU_0, a'\Omega r)$$

Velocity triangle:

$$\sin\phi = \frac{U_\infty(1-a)}{V_{rel}}, \quad \cos\phi = \frac{\Omega r(1+a')}{V_{rel}}$$

$$V_{rel}^2 = \frac{U_\infty^2(1-a)^2}{\sin^2\phi} = \frac{U_\infty(1-a)\Omega r(1+a')}{\sin\phi\cos\phi}$$

$$\frac{dT}{dr} = \frac{\rho B c U_0^2(1-a)^2}{2\sin^2\phi}\cdot C_n,$$

$$\frac{dM}{dr} = \frac{\rho B c U_0(1-a)\Omega r^2(1+a')}{2\sin\phi\cos\phi}\cdot C_t$$

Blade Element Momentum Model

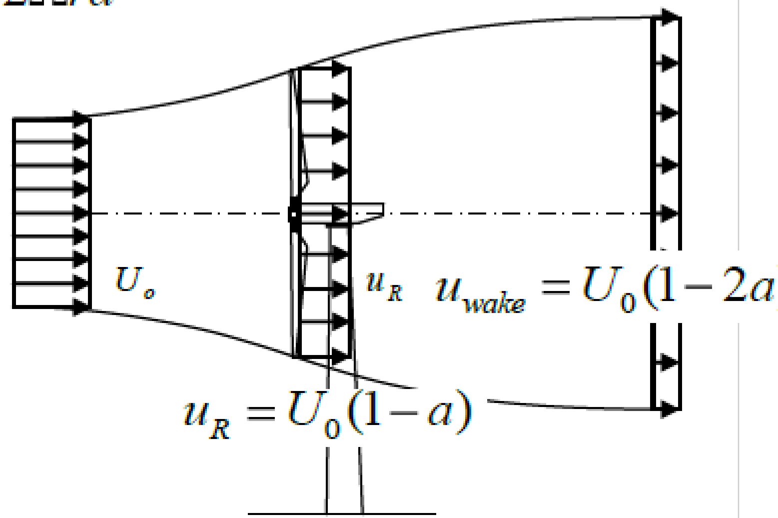
Induced velocities: $a = \dfrac{U_o - u_R}{U_o}$ $\qquad u_\theta = 2\Omega r a'$

Axial momentum balance:

$$\frac{dT}{dr} = \rho(U_0 - u_{wake})2\pi r u_R = 4\pi\rho U_0^2 a(1-a)$$

Moment of momentum balance:

$$\frac{dM}{dr} = \rho r u_\theta 2\pi r u_R = 4\pi\rho r^3 \Omega U_0 a'(1-a)$$



Combining blade element and and momentum expressions:

$$\boxed{\begin{aligned} a &= \frac{1}{4\sin^2\phi/(\sigma C_n)+1} \\ a' &= \frac{1}{4\sin\phi\cos\phi/(\sigma C_t)-1} \end{aligned}}$$

Solidity: $\sigma = Bc/2\pi r$

Blade Element Momentum Model

The Tip correction

The tip correction, F, corrects the axisymmetric approach to account for a finite number of blades:

$$\frac{dT}{dr} = 4\pi r \rho U_\infty^2 a F(1-a)$$

$$\frac{dM}{dr} = 4\pi \rho r^3 \Omega U_\infty a' F(1-a)$$

Prandtl tip correction formula:

$$F = \frac{2}{\pi}\cos^{-1}\left[\exp\left(-\frac{B(R-r)}{2r\sin\phi}\right)\right]$$

Modified expressions:

$$a = \frac{1}{4F\sin^2\phi/(\sigma C_n)+1}$$

$$a' = \frac{1}{4F\sin\phi\cos\phi/(\sigma C_t)-1}$$

Blade Element Momentum Model

Other corrections

Heavily loaded rotors: $C_T = \dfrac{dT}{\frac{1}{2}\rho U_\infty^2 2\pi r dr} = 4aF(1-a)$ **for a < 1/3**

$C_T = 4aF\left(1 - \dfrac{a}{4}(5-3a)\right)$ **for a > 1/3**

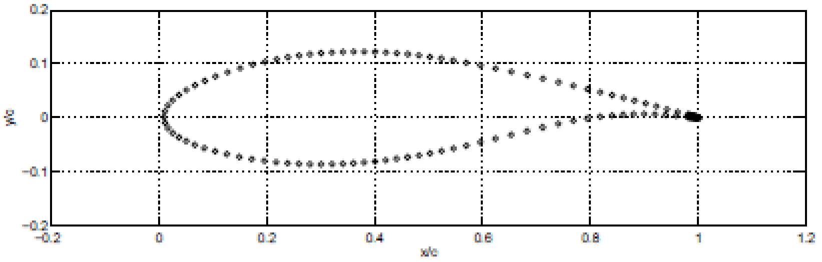
Yaw correction: $w_i = w_{i0}\left(1 + \dfrac{r}{R}\tan(\dfrac{\chi}{2})\cos(\theta_{blade} - \theta_0)\right)$

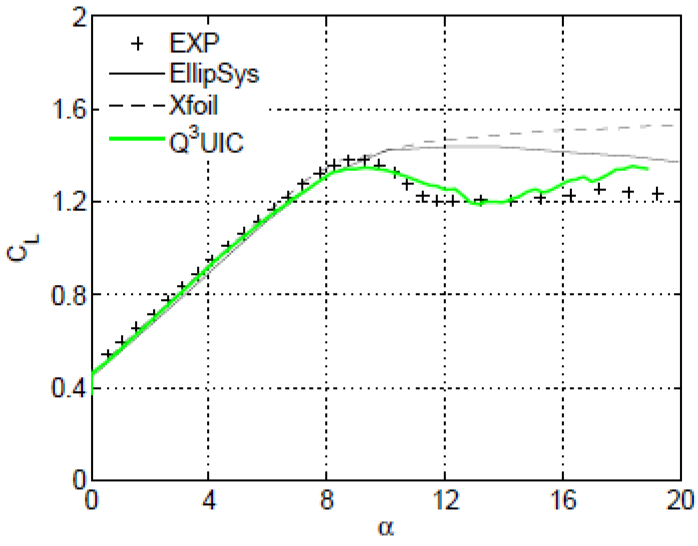
Dynamic wake: $Rf(r/R)\dfrac{du_i}{dt} + 4u_i(U_0 - u_i) = \dfrac{\Delta T}{2\pi r \Delta r}$

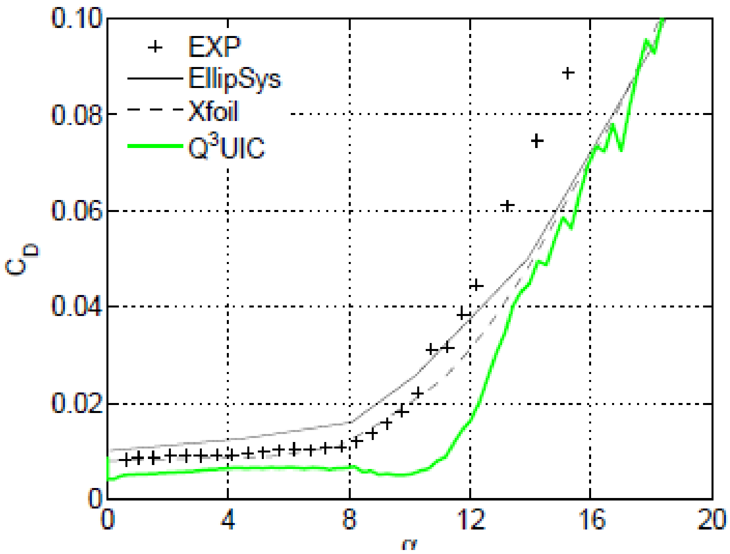
Blade Element Momentum Model

2D Airfoil data

DU-93-W-210



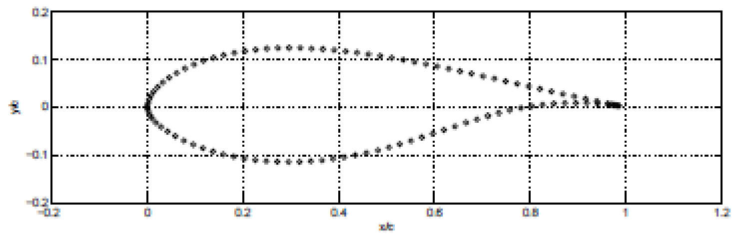


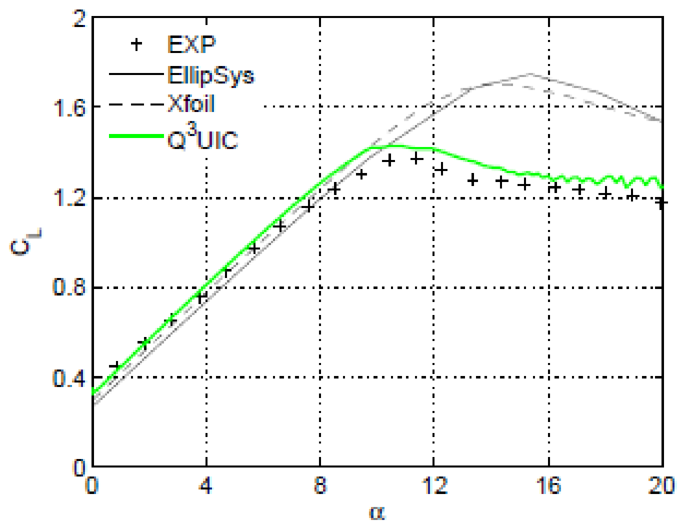


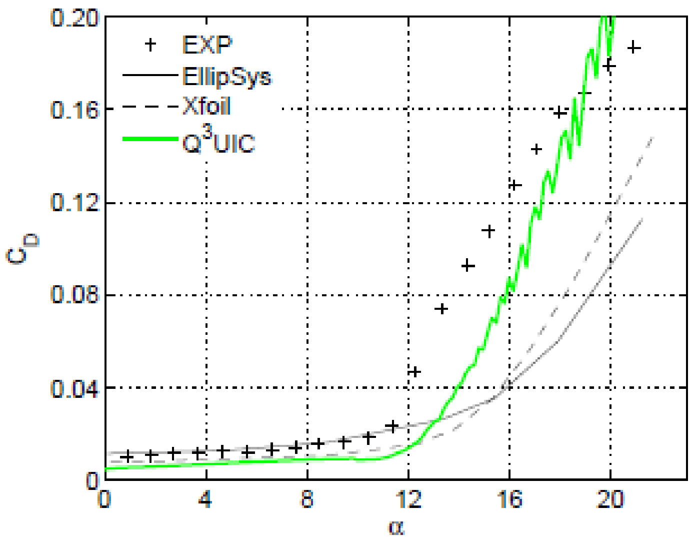
Blade Element Momentum Model

2D Airfoil data

FFA-W3-241







Blade Element Momentum Model

Suggested 3D corrections for airfoil data

Correction for rotational effects (AoA < 20 degrees):

$$C_{l,3D} = C_{l,2D} + a(c/r)^b \left[C_{l,inv} - C_{l,2D} \right]$$

Correction for finite rotor size (AoA > 45 degrees):

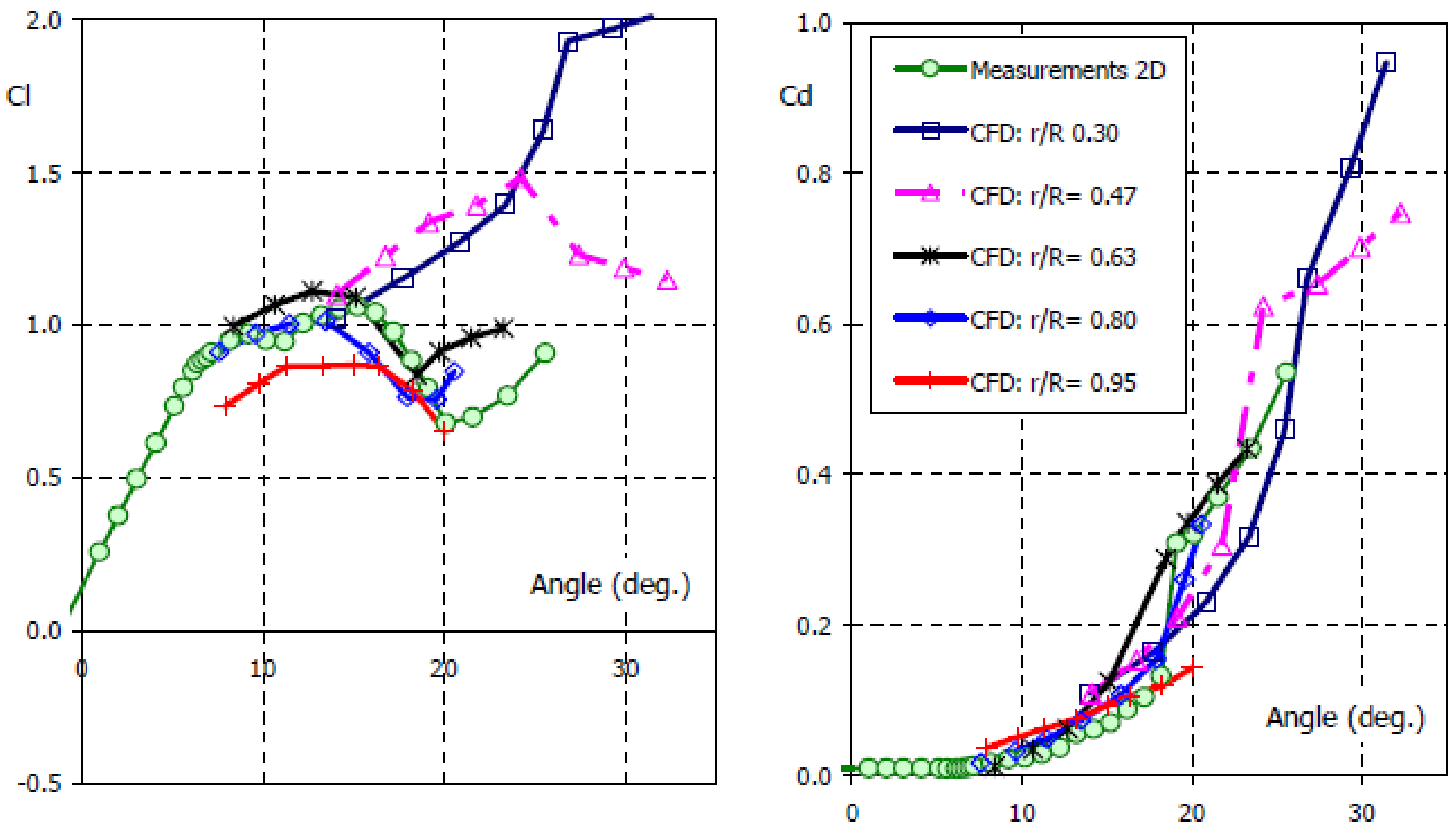
$$C_n = C_l \cos\phi + C_d \sin\phi = C_d(\alpha = 90^0)$$

$$C_t = C_l \sin\phi - C_d \cos\phi > 0$$

Linear interpolation for 20 degrees < AoA < 45 degrees

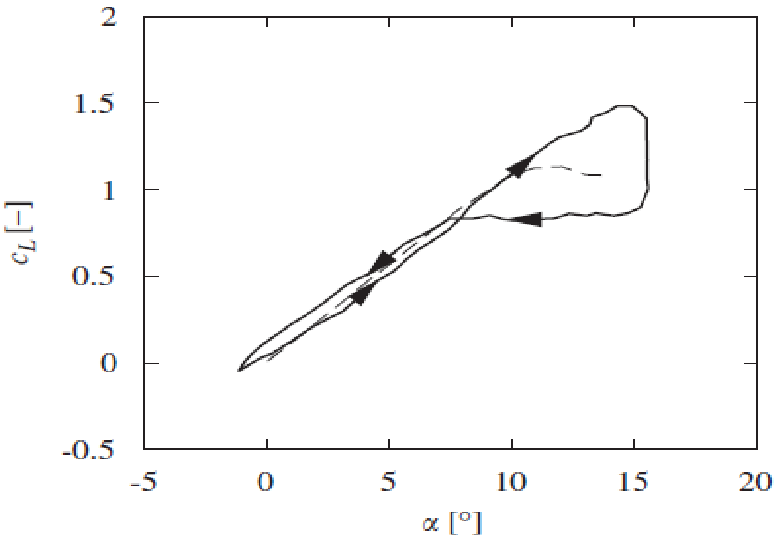
Blade Element Momentum Model

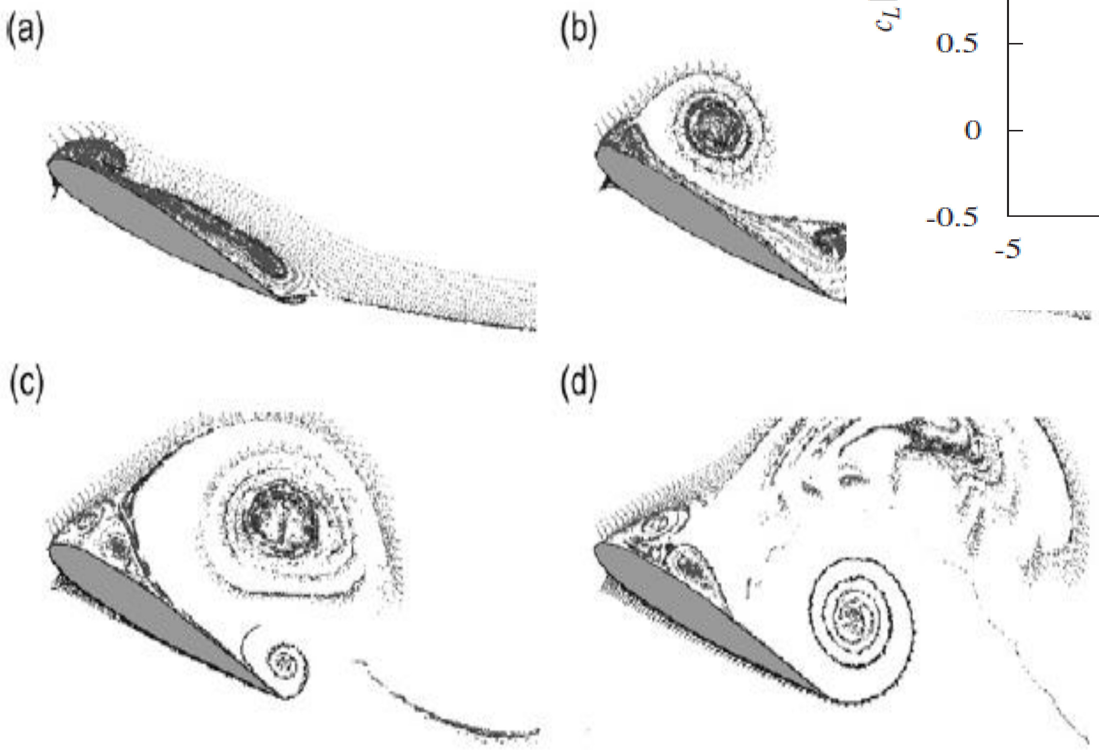
Computed 3D airfoil data



Blade Element Momentum Model

Dynamic stall



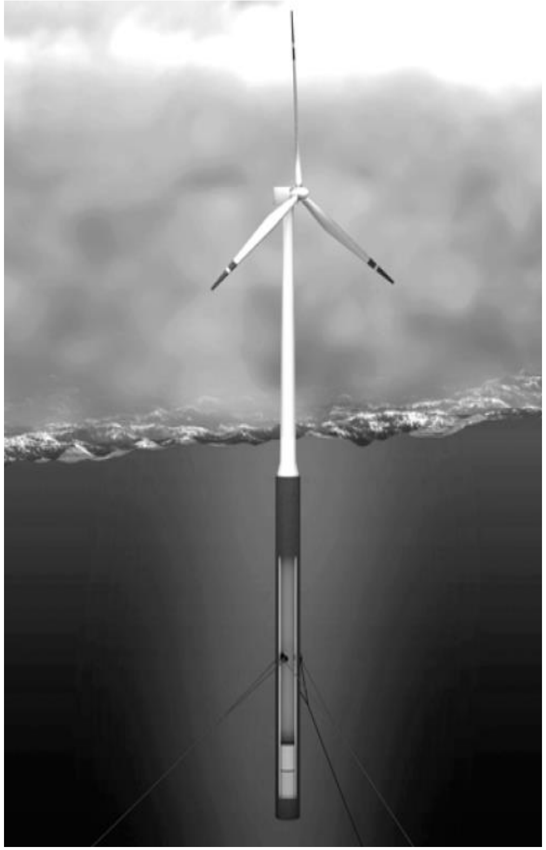


Blade Element Momentum Model

Dynamic stall

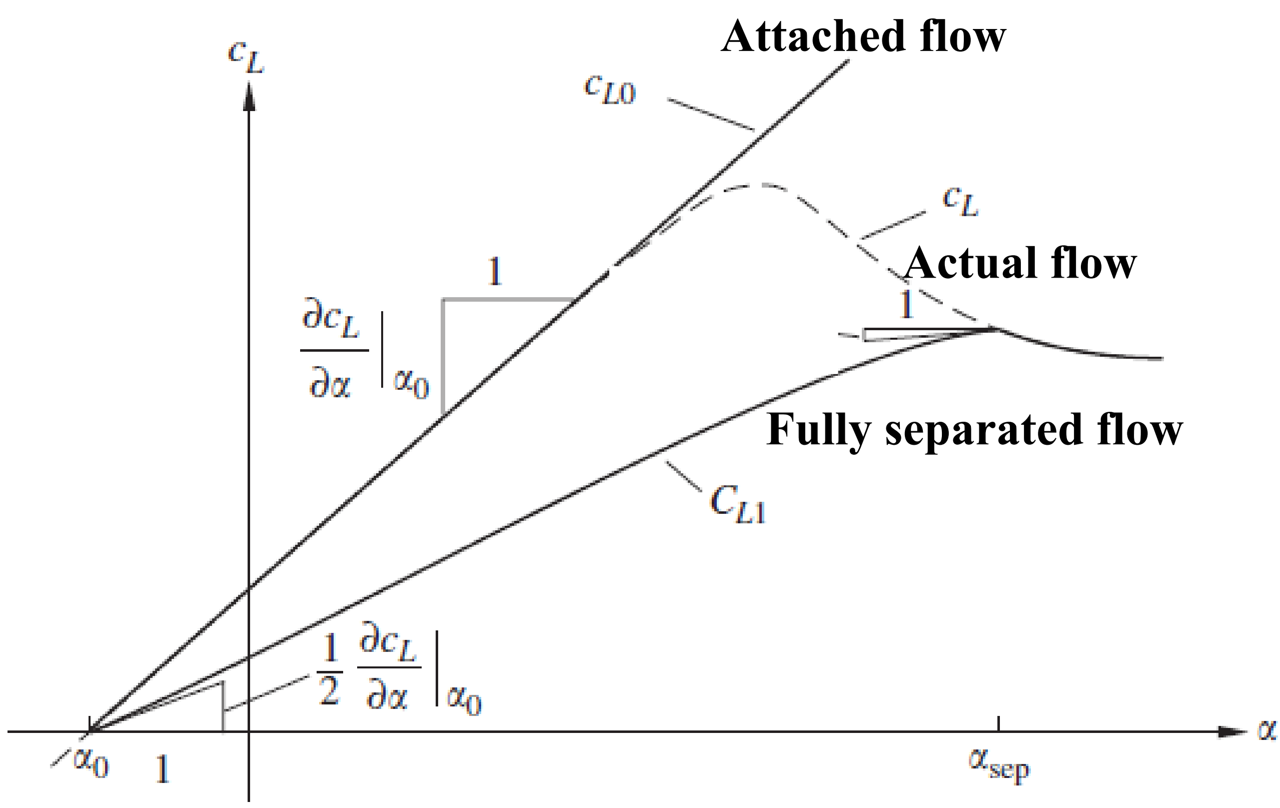
Dynamic stall is caused by:

- **Wind shear**
- **Atmospheric turbulence**
- **Tower shadow**
- **Rotors operating in yaw or tilt**
- **Dynamically deflected blades**
- **Turbine placed on floating structure**



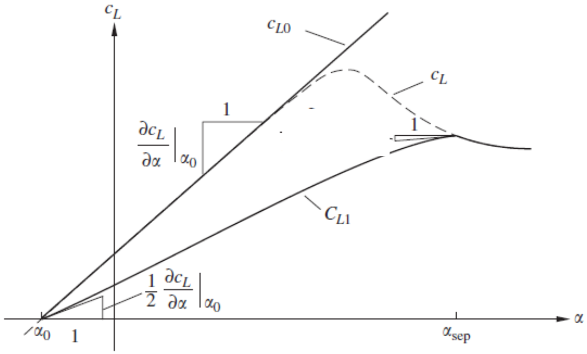
Blade Element Momentum Model

Dynamic stall model (Øye)



Blade Element Momentum Model

Dynamic stall model (Øye)



$f = 0$: **Fully separated**

$f = 1$: **Fully attached**

Linear interpolation:

$$C_L(f) = 2\pi(\alpha - \alpha_0)\cdot f + C_{L,sep}\cdot(1 - f) \Rightarrow$$

$$f_{static} = (C_{L,static} - C_{L,sep})/(2\pi(\alpha - \alpha_0) - C_{L,sep})$$

Dynamic approach:

$$\frac{df}{dt} = \frac{f_{static} - f}{\tau}$$

Final algorithm:

$$f_i = f_{static} + (f_{i-1} - f_{static})\cdot \exp\left(-\frac{\Delta t}{\tau}\right)$$

Conclusions and further work

- Today, Wind Turbine Aerodynamics represents an important part of the research in the fluid mechanical field
- The range of studies spans from basic instability analysis to control and design of wind turbines, and from micro-thin airfoil boundary layers to km long wind farms and ABLs.
- Developments and research are carried out in both the 'classical' aerodynamic topics as well as in the latest development in computational techniques
- The grand challenge in wind turbine aerodynamics is shared with most of the fluid dynamical community; namely Turbulence and Laminar-Turbulent Transition
- Both development in computational techniques (CFD) as well as more experiments are needed; not only for validation, but also to get new insight and inspiration for new ideas

If you wish to read more

Jens N. Sørensen:
'General Momentum Theory for Horizontal Axis Wind Turbines'. Springer, 2016.

Sorry for this inappropriate commercial break

Any Questions?